ON THE DIFFERENCE SENSIBILITY FOR THE VALUATION OF SPACE DISTANCES WITH THE HELP OF ARM MOVEMENTS.[1]

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The experiments which form the basis of this paper, were commenced in the institute for experimental psychology here under Professor Wundt, in the summer semester of 1892, and continued during the following two semesters. The object was to determine the fineness of the space-sense as shown by the difference sensibility for movements of the arm in vertical directions upwards and downwards, at different heights, standards and rates of movement. The influence of the leaving of the end points determined or undetermined is also considered. The experiments consider otherwise different asymmetrical influences of the body on the valuations and certain phenomena which result from the loading of the moved members with weights. Either the two arms together or only one were moved as far as possible without bending of the elbow and hand-joints. There are two chief parts to the experiments. The first comprises such as were made by drawing simple curves upon a vertically standing table or on the wall with a soft lead pencil. These are described as the pencil experiments. They are one-handed or two-handed, according as the movements belonging to a single experiment were made with only one arm or with the two arms. In both cases the direction of the arms was forwards, parallel to the median and at right angles to the side plane of the body. The second part comprises such one-handed experiments as were made by what is here described as the angle method. For this purpose a vertically standing upwards and downwards movable table was used. On one side of this, the side adjacent to the arm, a half circle was described, whose radius was 68 cm. and at every one-fourth degree of it a hole was bored through. By this means movements of different extents in angles could be marked off by the placing of plugs in the holes. The observer sat on a chair, and the centre of the circle could be made to correspond to the centre of the arm's motion by shifting of the table up and down.

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[1] Translation in part of the original as written in German, spring and summer of 1893. See Wundt, *Grundzü.*, 4th Ed. Vol. I. p. 429. *Revue Philosoph.* Dec. 1893, p. 663. To all of those who acted as observers, I am indebted for their patience, namely, Geh. Prof. Wundt, Dr. Külpe, Dr. Kirschmann, Dr. Cohn, Mr. Kiesow, Mr. Rostosky, Mr. Child, Mr. Rogers and Mr. Hicks, as well as to the first mentioned for the conveniences of his laboratory.

I.

Experiments by Means of the Pencil Method.

The method of relative valuation and the method of relative difference sensibility.

When one investigates the difference sensibility for motion sensations, he can either leave the extent of the normal distances in any series or group of experiments determined, or he can allow the observer to fix the extent himself. In the experiments of Fullerton and Cattel[1] both methods are used. In the latter case the psychophysical methods of the average error, the mean valuation and the right and wrong cases can be used. Where, however, the normal extents, as also the compared extents, are fixed, the method of just perceptible change, the method of mean valuation and the method of right and wrong cases can be used. The fixing of the normal and compared extents brings about groups of other sensations, which complicate the judging of the pure motion sensations. These are contact, pressure and resistance sensations. Hence it is desirable to have the movements made without contact, pressure or resistance through the fixing of the extents. If, also, only the normal extents are fixed, the pressure contact and resistance at the end can make these of no use as standards for the compared extents. If one investigates the pure motion sensations, then one must, if possible, make the movements without fixing the end points. This can only take place if one makes use of shadows, photography, mirrors, or some other such optical apparatus. My first group of experiments made by the pencil method give impure motion sensations in so far as contact sensations are felt with these. Their calculation, however, is similar to such as can be made without contact. In all such movements, where the making of the standard curves equal to one another in any single series is dependent on the observer, there exists a variation of the individual from the average normal length. The influence of this variation upon the average differences between normal and compared extents where one uses the method of average error, is considered in another place.[2] Between the smallest and largest normal extents in any single series, one can further reckon out whether the individual differences upon increase of their respective normal extents increase or decrease. If we call such of the individual differences between the normal and compared extents, where the latter are greater than the former, plus; where they are smaller, minus; and where they are equal to, equal,—then the individual differences in any series can either be all plus and equal, all minus and equal, or also there can be all plus, minus and equal differences. The difference sensibility where the normal extents increased, would increase or decrease respectively with increase and decrease in the individual differences in proportion to the increase in their normal extents. If they are all minus, or minus and equal, then the difference sensibility upon increase in the normal extents would respectively increase or decrease in proportion to the increase in their normal extents inversely with the increase or decrease of their respective differences. Where these are all either plus, minus and equal, one must consider the cases where they are plus and equal and where they are minus and equal, each by itself, and from this twofold result reckon out the mean difference sensibility. The method by which the relative

[1] "The Perception of Small Differences." Fullerton and Cattel. Philadelphia, 1892.
[2] Pages 375, 379, par. 12, 381, 391d.

difference sensibility with the variations in the normal curves, where all the individual differences are plus, minus or both plus and minus, is reckoned, I call the method of relative difference sensibility. The reckoning of the difference sensibility by this method is very similar to the reckoning used in the method of relative valuation. If we let the individual differences in a series be all plus, all minus, or both plus and minus, then the valuation (judging) will increase where the compared extents increase with increase in their normal extents. This will be shown by the absolute increase of the individual differences with increase in their respective normal extents. Inversely, the valuation will decrease where the compared extents decrease with the increase in their normal distances. This will be shown by the absolute decrease in the individual differences with increase in their respective normal distances. The method by which this valuation within the variations of the normal extents in a series is reckoned out, I call the method of relative valuation. The estimation of this valuation is similar to the estimation of the difference sensibility by the method of relative sensibility, where the individual differences are either all plus or all minus. As it is the simplest, I give at first the reckoning out of the method of relative valuation.

Description of the Reckoning Out of the Method of Relative Valuation.

We let m_1 m_2, m_3, etc. . . . . m_x be the simple digits 1, 2, 3, etc. . . . . x, of which m_1 represents the greatest normal extent, m_2 the one in the same series which is just smaller than m_1, and so on up to $m_x = x$, which represents the smallest normal extent. We let further n_x, n_{x-1}, n_{x-2}, etc. . . . . n_3, n_2, n_1, be the same digits x, $x-1$, $x-2$ . . . . 3, 2, 1, of which, however, $n_x = x$ represents the smallest difference between any individual normal extent and its compared extent in the same series, $n_{x-1} = x-1$, that which is somewhat greater than n_x up to $n_1 = 1$, which represents the greatest difference between any normal extent and its compared extent in the series. We add, then, each m value to its respective n value, so that the m value, which represents any individual normal extent, is added to its respective n value, which represents the simple difference between this normal extent and its compared extent. r, o, s, etc. . . . . g are the sums of such individual values as $m_1 + n_x$, etc. . . . . $m_x + n_1$, according as the valuation takes place. In every series there exists x such values. d, e, etc. . . . f are the variations of these values from $(m_n + 1) = (n_1 + 1)$. q represents the average of these variations. If m_1, m_2, etc. . . . . m_x are added respectively to n_x, n_{x-1}, etc. . . . . n_1, then will d, e, etc. . . . . f each be like null and the average variation q will also be like null. If they are added together inversely, namely m_1 to n_1, m_2 to n_2, etc. . . . . m_x to n_x, then the average variation will be Q. This Q gives us the greatest possible average variations $\frac{q}{Q} \times {}^{100}$ gives us a value which shows whether the differences between the normal and compared extents increase or decrease with the increase in their normal extents, *i. e.*, whether the valuation increases or decreases with increase in the normal extents. If $\frac{q}{Q} \times {}^{100}$ is equal to 100, *i. e.*, $q = Q$, then in comparison with the valuation of the smallest compared extent, a gradual increase of the compared extent with reference to its normal extent would be shown with increase of

this last in the same series. If $\frac{q}{Q} \times {}^{100}$ is equal to 0, *i. e.*, $q = 0$, then a similar gradual decrease would be shown. If $\frac{q}{Q} \times {}^{100}$ is equal to 50, then there would be shown on the average for the series neither such a decrease nor such an increase. If the value of $\frac{q}{Q} \times {}^{100}$ lay between 0 and 50, then there would be more or less of a decrease; if between 50 and 100, more or less of an increase.

Description of the Reckoning Out of the Method of Relative Difference Sensibility.

If all the individual differences between the normal and compared extents are plus, then the difference sensibility, where the value obtained by the method of relative valuation is 0, increases with increase in the normal extents; where 100, decreases; where 50, it neither increases nor decreases on the average; where it is between 0 and 50, it more or less increases, and where between 50 and 100, it more or less decreases. If the individual differences are both plus and minus, or equal and minus, then it is necessary to reckon out the value obtained by the method of relative valuation for the different normal extents which show plus or plus and equal differences between the normal and compared extents, and likewise for the different normal extents which show minus or minus and equal differences. The reckoning out of this latter will, however, be different from the reckoning in the case of the method of relative valuation only in so far as one allows $n_x = x$ to represent the difference, which is nearest the value null, $n_x - 1 = x - 1$, that which is somewhat smaller, and $n_1 = 1$, the minus difference, which is most unequal to the simple difference null. The values of q, Q, and $\frac{q}{Q} \times {}^{100}$ are now reckoned out, and also the mean of the two values, namely, that obtained from the normal extents which give plus differences and that obtained from those which give minus. If this value is null, then the difference sensibility gradually increases with increase in the normal extents; if it is 100 it gradually decreases; if it is 50, then it neither increases nor decreases on the average; if it is between 0 and 50 it increases more or less, and if between 50 and 100 it decreases more or less.

It is here to be remarked that when we speak of the normal extents in any single series, from the standpoint of difference sensibility, we have to do with an absolute increase or decrease in the series and not with a relative. Only when we compare this increase and decrease in one series with the same in another can we speak of a relative difference sensibility. If one wished to estimate the relative difference sensibility for each series alone, then instead of using the individual differences to reckon out the n values, he must use these divided by their respective individual differences. This, however, has little value. In the same way, we can only speak about an absolute increase or decrease in the valuation, unless we use, in reckoning out the n values in the method of relative valuation, instead of the individual differences, these divided by their respective normal extents. This has also, however, insufficient value. We can, however, speak about a relative valuation or a relative difference sensibility in connection with the values obtained by these methods, in so far as in the case of the first method one takes the valuation where the normal extent is smallest, as normal valuation for the series, and in the case of the other, the difference sensibility

as shown, where the individual difference is null, or nearest above or below null, as the normal difference sensibility in each series. The values obtained by these methods are dependent on this normal valuation and on this normal difference sensibility, and to some extent show a relative valuation and a relative difference sensibility. From these considerations we obtain four methods which show connections between the individual series and the increase in the normal extents, as well as the increase or decrease connected with this in the increase and decrease in the valuation and in the difference sensibility, namely, the two which we have above completely described, the method of relative valuation and the method of relative difference sensibility, and two others which correspond to these, and which give values for the increase and decrease in the relative valuation and the relative difference sensibility in the individual series, instead of the absolute valuation and the absolute difference sensibility, in the individual normal extents. These two latter methods we do not consider further.

The reckoning out of the method of relative valuation and of the method of relative difference sensibility is accompanied by certain difficulties which can be overcome only through carefully choosing the number of experiments and the digits. In the first place, if there is more than a single normal extent, of the same value, one must take the mean of these and also of their respective differences and reckon out the results in the ordinary way with this reduced number of experiments in the series. The same can also be done where there is more than a single simple difference in the same series of the same value. In the second place the average value "q" might be the seat of an error in that the individual parts of V $(m_x + 1)$ work contrary to one another. A table (Part II) with an instance where this occurs is given below, where 2.40 is $= q = Q$.

Part I.			Part II.					
$M + N$	= sum	$V(m_x + 1)$	N L	D	M	N	M+N	$V(m_x + 1)$
m_1+n_x	r	d	20.50	— .50	3	1	4	2
$m_2+n_{(x-1)}$	o	e	21.50	—1.50	1	2	3	3
$m_3+n_{(x-2)}$	s	etc.	21.25	—2.00	2	3	5	1
etc.		etc.	18.25	—3.75	4	4	8	2
m_x+n_1	g	f	17.50	—4.00	5	5	10	4
	5 | r + etc. + g / m_x+1	5 | d + e + etc. + f / q						5 | 12 / 2.40

But it is easy to see that it should be smaller. This error is not frequent and it would serve no present purpose to reckon it out, in so far as no wholly exact value is sought, but only a relative one.

The method of relative valuation and the method of relative dif-

ference sensibility will be used as an aid in the reckoning out of the difference sensibility in the experiments made by means of the pencil method, and is represented by the symbols M. R. V. and M. D. S.

A.

The One-Handed Movements.

1. A General Consideration of the First Experiments.

I have reckoned out these experiments all together, without taking into consideration the length of the curves. The results show that out of a total number of experiments of 1008, $1\frac{3}{4}$ per cent. were overvalued, $\frac{1}{8}$ per cent. were valued correctly, and $18\frac{1}{8}$ per cent. were undervalued. Each of the sixty-six series was at first treated separately. The average differences of the experiments, where over, under and equal valuations took place, along with the average variation of these, were reckoned out for all the experiments. These were then grouped into combinations of six and seven series in a group. These groups were then calculated all together. The average of the whole of the averages of the normal and compared curves, of their differences, of their variations, of the experiments of the sixty-six series, was then reckoned out. In each of the sixty-six series there were overvalued experiments, and in fifty-one of them there were experiments where undervaluations took place. The number of experiments, where correct valuations took place, was so small that we shall not take them into consideration here. The lengths of the normal curves varied between 10 and 60 cm. The average of all the normal curves, where an overvaluation took place, is 22.31 cm. The same, where an undervaluation took place, is 29.62 cm. The average of the differences of the former is 5.48 cm., of the latter, 3.53 cm. When both the normal and compared curves are added together, the average obtained in the first place was 25.96 cm., and in the second, 26.93 cm. In this way one obtains an average difference in valuation of .97 cm. The average of all of the average normal curves of each series, without consideration of the over and undervaluation, is 23.59 cm. The arithmetical average of all the average differences was 2.75 cm. overvaluation, an average difference sensibility of about $\frac{1}{10}$. It is seen, then, that both according to the number of experiments, and also to the average differences, a significant overvaluation had taken place. A review of the series shows that nineteen series had an average normal curve of between 10 and 15 cm., twenty of between 20 and 30 cm., fifteen of between 30 and 40 cm., four of between 40 and 50 cm. and two of between 50 and 60 cm. The average of the average variations of fifty-one series, where an overvaluation took place, was 2.37 cm. For thirty-seven series, where an undervaluation took place, it was 1.00 cm. As this was small in such a general consideration, I have not taken it into account.

It is desirable now to find out what sort of influences the upwards and downwards direction of the motions, the changes in the normal lengths and in the height of the starting points, as well as special differences in the observers and in the methods of experimenting, could show. In the case of the first fifteen series the height of the starting point was not taken, and we do not consider these further. Instead of a lead pencil being used, also a small brush with lamp black mixture was used. Of the remaining fifty-one series, thirty were made in the upwards direction[1] and twenty-one in the

[1] See table II. p. 407.

downwards.[1] The accompanying tables[1] show the heights of the starting points from the lowest, which is marked with minus, to the highest, which is marked with plus. The horizontal starting point, which could be only roughly estimated, is marked with zero.

In considering this first group of experiments it is important to remark that they were all made more with respect to the convenience of the observer than to any previously determined plan of experimenting. The beats of the metronome, which regulated the rate of movement as well as the length of the curves, show also to some extent the disposition of the observer apart from the exactness of his judging. Very quick time and very slow time was disagreeable. A look at the diagrams[2] shows also that the average lengths of the normal curves for the series varied more under the horizontal point in the case of the upwards movements, but in the case of the downwards movements more over the horizontal point. Not only also are the limits for the average normal lengths greater in these directions, but the number of series is also greater in the case of upwards movements under the horizontal point and in the case of downwards movements above the same. Apart from series 20 of the upwards movements and series 17 of the downwards, a certain attempt was made to make all the normal curves in each series as much as possible equal to one another. There was nothing binding in this, however, as it was thought that it could possibly disturb the power of valuing correctly between the normal and compared curves. Still a certain indirect valuation in this respect was present in each series. Each new normal curve was more or less of an attempt to reproduce the former. In so far, such a valuation was implicated. The variation of the normal curves gives a value for this. In the case of the upwards motions, the changes in the variations follow the changes in the lengths of the average normal lengths, with few exceptions. In the case of the downwards motions this is also very often the case. This shows that the difference sensibility is dependent on the length of the normal curves. In many cases also, the normal curves were made of the same length as the just previously described compared curves. In such cases the compared curves were made smaller than their normal curves. These two pairs of curves appear, then, to form a separate basis of valuation. The above described average values of all the series can be considered as a sum of such valuations, of which the average differences show the exactness of the valuation. As the average overvaluation of the experiments of all the 66 series, where overvaluation took place, is 5.48 cm., the corresponding average undervaluation 3.53 cm., and the average overvaluation in all the experiments was 2.75 cm., we see that the difference sensibility would amount to about $\frac{1}{20}$. +23.59 cm. is the average normal length of all the experiments; $\frac{2.75}{23.59 \times 2} = \frac{1}{20} =$ difference sensibility. In general, the curves show no very exact changes caused by changes in the lengths of the normal curves and in the starting points. The larger normal curves show in the case of the upwards movements, where the starting points were, —30 and —60 cm. larger average differences than do the smaller normal curves. In the case of the downwards movements, the average differences show the opposite results, so that the smaller normal curves are more overvalued than the larger. It is easy to see that on account of the changes in the average normal lengths and in their variations, the influence of the changes in the starting points in these experiments can not be

[1]See table I. p. 406.
[2]See original.

very exactly determined. Where also the variations of the average normal curves and the changes of the starting points do not vary much, the variations of the average differences are too great to give exact results. In order to find out the influence of the starting points, I have made the two-handed experiments, which are given below (B), and I have also made other additional one-handed experiments to determine the influence of the extent of the normal curves. I give in addition certain remarks on the individual series. By the method of relative valuation above described, I have reckoned out in the individual series the tendencies towards under and overvaluation. From this it appears that in the case of upwards motions, where the starting points are very low or very high, an undervaluation takes place more and more with the increase in the length of the normal curves. Under and at the horizontal point, however, more of an overvaluation takes place. These overvaluations and undervaluations are not very much dependent either on the changes in the average normal lengths nor on the changes in the variations of the individual normal curves from their average normal lengths. In the case of the downwards motions with low starting points, a distinct undervaluation of a similar kind takes place, and is also present where the starting points are taken quite high. Only when the starting points are very high is an overvaluation met with.

2. *Consideration of the Individual Series.*

a. Series 11 and 12 of the upwards motions and series 8 of the downwards were made while the observer was seated. They show no special deviations from the other series. Among the later experiments are some which show more particularly differences between the standing and sitting posture.

b. With and without the beating of the metronome. Series 16, 24, 26 and 5 of the upwards motions were carried out with no accompanying beating of the metronome. It is to be remarked that the variation of the normal curves in each case is very small. The angle method experiments to be described later will show the more important influence of the rate of movement upon the relative valuation. In general it appears that the observer without the beating of the metronome, chooses the most convenient rate of movement for the starting point and length of movement. The metronome's beating has less influence on the variation of the normal lengths with such as are accustomed to see the regular motions of machines, or to make regular motions themselves. In these cases the two groups of sensations, the normal and the compared, are more or less localized in the organs of motion. The motions are automatically made. Where this is not the case the metronome exercises a greater and more distinct influence upon the variation in the normal curves. Along with this the two groups of motion sensations can be considered more as sensations of the willing impulse, which are localized in the glottis and in the breathing apparatus. As the metronome tends to make these motions also automatic, however, these sensations are not so important (as materials in investigating) as in either case where one makes movements of unusual length and height.

c. Influence of the lengths of the radii. The irregularity of many of the average differences which are dependent neither upon the differences in the variations of the normal curves, nor upon the differences in the heights, can be very well ascribed to the bending

of the arm. It is desirable, therefore, to find out what relation the radii of the normal curves bear to the radii of the compared ones. The curves were more or less bent towards the extremities. The measuring of the radii, then, was confined to the normal arcs in the middle of the curves. A large wooden compass was used to measure these directly. Where the radii of the normal curves are smaller than those of the compared, the differences are marked with plus, where larger with minus. In the case of the upwards motions, it appears that the normal radii are smaller where the starting points are very high and very low than where the points lie between these extremes. It is meant that the variations in the lengths of the normal radii and the average differences between those of the normal and compared, show the value of this measuring. Where one variation is relatively small, so is the other, where large, likewise. From this it follows that there existed regular fluctuations in the bending of the arm. The bending of the arm alternates more or less in each series, and the average differences of the radii show, then, how the valuations of the movements are influenced through this.

If we consider now more particularly series 2, 5, 6, 7 and 9 of the upwards movements, which have starting points very nearly alike, we find that by increasing the height of the starting point as well as by lengthening the average lengths of the normal curves, the lengths of the average radii of these increase. If we consider, however, only the changes of the lengths of the average differences of the radii in connection with the changes in the average differences of the curves, then we find that they stand in a directly inverse relation to one another. The exception in series 6 can be very easily ascribed to the greatness of the variation.

In series 1, 5, 6 and 11 of the downwards motions, the lengths of the average normal radii increase with the heightening of the starting points. The first three series were made under the horizontal point and can be considered by themselves. The average differences of the normal and compared curves stand in a directly inverse relation to the average differences of their radii. Apart from the starting point the increase in the average lengths of the curves is accompanied by a decrease in the average differences of the normal and compared curves, and by an increase in the average differences of the radii themselves.

In the case of both upwards and downwards movements, then, it results that an increase in the average differences of the normal and compared curves is accompanied by a decrease in the lengths of the differences of their radii. The greater normal curves are, however, more overvalued in the former case than in the latter.

d. What relation do the curves bear to their corresponding angles and perpendiculars? I have drawn diagrams[2], which give the average values for the above described series. For series 2, 5, 6, 7 and 9 of the upwards motions, the average of the heights of the starting points was —52.10 cm. The average of the average normal curves is 26.67 cm.; of the compared 34.10 cm.; of the average radii of the first 56.72 cm., of the last 61.96 cm. In the case of the downwards movements, the average of the heights of the starting points for series 1, 5 and 6 was —42.87 cm. The average of the average normal curves is 16.96 cm.; of the average compared 17.65 cm.; of the radii of the first 54.19 cm., of the last 56.63 cm. From these values I have reckoned out the corresponding angles and the perpendiculars to the horizontal plane lying between the extremities of the curves. Dr. J. Loeb[1] has shown

[1]Archiv für die ge. Phy. d. M. u. d. Th. von Pflüger. Bd. 46, I. S. 1; Bd. 41, S. 107.
[2]See original.

that where the starting points lie under the horizontal plane, motions made at right angles to this plane show great deviations between the normal and compared distances, and this just so much the more that the starting points are distant from the horizontal plane. If the observer had described such straight lines without changing the angles of movement, the above perpendiculars would show my results to agree with those of Loeb's. My experiments show, however, that in order to make lines of equal greatness, the angles at the shoulder should be taken as the standard and not the perpendiculars. In both cases, whether one makes movements with the arm held straight out in the form of curves or with the elbow joint bent in the form of straight lines, it is to be assumed that the mass of sensations are so nearly equal to one another that they do not need separate consideration. There exists a probability for this in so far as the contact and holding of the elbow joint when the arm is fully stretched out, compensate for the other kinds of stimuli which arise through the bending of the elbow joints and the muscles. How far the sensations due to the moving muscles and to the so-called will-impulse are to be taken as the basis for the valuation, will be explained later. The chief point to be noted here is that the angle at the shoulder joint (or joints) is to be taken as the basis for the valuation of the distance moved. The above described changes in the perpendiculars bear a similar relation to their corresponding angles at the shoulder in the case of the upwards movements to what 8.12 cm. and 26.42 cm. bear to 26° 56′ and 31° 34′, and in the case of downwards movements to what 9.25 cm. and 5.72 cm. bear to 15° 53′ and 17° 48′. These two accompanying diagrams[1] give us examples as to how these perpendiculars stand to their curves or angles at the shoulder. This relation changes with the starting points. In the horizontal plane both stand in a very close relation to one another. High above and low below, the greatest deviation between the two occurs. Loeb used the two arms in his experiments. The normal arm had always the horizontal plane as starting point, while the starting point for the other arm was varied.

3. The Later One-Handed Experiments of the Winter Semester, 1892-3.

As above mentioned I have made further experiments with special respect to the changes in the average differences which arise through changes in the lengths of the normal curves and in the heights of the starting points. The latter I have considered more especially in the two-handed experiments to be described later. The experiments we consider here, consisting of ninety-nine series, were all one-handed. In twenty-nine series the starting points were changed. In all the ninety-nine series an attempt was made as much as possible to keep the variation of the normal curves for each series small. The number of experiments for each series was ten. The observers were Mr. Child, "C.," and the experimenter himself, "S." The former was seated, while the latter remained standing and acted as his own experimenter. The horizontal plane for the former was 102.5 cm. above the floor, and for the latter 130 cm. above the same. The number of series made by the first was nine in the upwards direction and eleven in the downwards; by the last, forty-five in the former and thirty-four in the latter. The beating of the metronome was, with the exception of eleven series, by observer S., where it made 180 beats in the minute, and five others where it made 120, kept at sixty beats to the minute.

[1] See the original.

(1.) *Observer C.*—The series by observer C. were made, with two exceptions, in order to find out the changes in the average differences caused by changes in the lengths of the average normal curves. In general the average differences were not nearly as large as in the earlier experiments. The variation in the normal curves follow in their relative values the changes in the lengths of the average normal curves. Where the latter decrease, so do the former. With the same decrease the average differences in the case of the downwards motions also decrease, and inversely in each individual series the increase in the individual differences is accompanied by an increase in the lengths of the normal curves.

Upwards movements. The accompanying tables[1] show the two groups of series of upwards motions by observer C. Series 4, group II., had, besides the starting point in the horizontal plane for the normal curves, a different one for the compared. It is, then, left out of consideration here. The question presents itself here, "How does the relative and absolute difference sensibility decrease or increase with the increase in the lengths of the normal curves?" The variations of the normal curves and the differences of the normal and compared curves can only be considered as absolute spatial changes, which serve as a basis for an absolute difference sensibility. With the increase in the normal curves they also increase. Hence the absolute difference sensibility decreases. The M. R. V. shows that where the individual differences in each series are considered, each increase in the normal curves, where these are greatest, is accompanied by a decrease in the individual differences; where they are smallest by an increase, however. The average differences show that with increase in the average normal curves the average differences increase in group II., and inversely in group I. decrease. Were the average differences not reckoned out, one could estimate the difference sensibility to a certain extent by the method of relative difference sensibility. The figures obtained by this method show that in group I. the difference sensibility increases with the increase in the individual normal curves, and this just in proportion as the individual normal curves decrease. In group II., however, it decreases, and just in proportion as the normal curves increase.

Downwards movements. Two groups of experiments were also made by observer C in the downwards direction. The M. R. V. shows a constant increase of the absolute values of the individual differences in the series with the decrease in the normal curves, and this the more the average normal curves increase. The average differences in the two groups are themselves different. In series 1-4, group II., they are greater than in series 1-8, group I. The difference sensibility is then inversely greater with the latter. In both groups the increase in the lengths of the average normal curves appears to cause neither an increase nor a decrease in their average differences. They do not allow of a judgment in regard to the absolute difference sensibility. In both groups the norm al variations decrease also with decrease in the average normal curves. As above mentioned there is ground here for concluding that the absolute difference sensibility decreases with the decrease in the lengths of the average normal curves.

Changes in the starting points. The two remaining series of downwards movements of observer C. show with increase of the starting points a greater decrease of the compared curves in relation to their normal. The absolute difference sensibility was great-

[1] See the original.

er above than below, because the differences in both cases were greater. This agrees with what was said in regard to the other downwards movements.

(2.) *Observer S.*—Upwards movements. The six groups of upwards movements of observer S. were made under different conditions. In all of these, apart from group V., the variations were greater than where the average normal curves were greater, and smaller where these were smaller. Hence the difference sensibility decreases with increase of the normal curves. The lengths of the averages of these vary between 5.95 cm. and 53.95 cm. The average differences in group I. show no regular increase or decrease, with the increase in the lengths of the average normal curves. The average of the last three series is greater than that of the first three. This shows a finer difference sensibility for the greater curves than for the smaller. The dividing of the average of the compared curves by the same of the normal ones shows the difference sensibility from 5.95 cm. to 23.10 cm. to gradually increase. At 30.15 cm. it is again less and at 44.70 cm. is about the same as at 23.10 cm. The M. D. S. shows that the difference sensibility, as shown within the limits of a group, increases on the average with increase in the individual normal curves.

Group II. of observer S. was different to group I. only in so far as the metronome beat 180 times in the minute instead of sixty. By this the rate of the movement was considerably increased. In general an overvaluation took place, and this is greater with the larger normal curves and smaller with the smaller. The variations of the normal curves are also greater with the greater average normal curves and smaller with the smaller. Series 3 forms an exception to this. The normal variation of the same is greater than the normal variation in series 2. From the standpoint of the average differences and of the average variations of the normal curves, there appears a greater absolute difference sensibility when the normal curves are smaller than when they are larger. The M. R. V. shows in general in the individual series a decrease in the individual differences with increase in the individual normal curves. This decrease is somewhat larger when the normal curves are greater than when they are smaller. In series 2, 4 and 5, the M. D. S. shows that the difference sensibility in the average decreases with increase in the individual normal curves, while in series 1 and 3 it decreases.

Group III. was made in such a way that in each series four curves of different normal lengths were made after one another in such a way that the second was greater than the first, the third greater than the second, and the fourth greater than the third. Each time it was attempted in the following sixteen series to repeat these four experiments with similar normal lengths. For the first twelve of these, the metronome beat sixty times in the minute, and for the last five (group V.) 120 times to the minute. In groups III. and V. the variations are relatively greater than in the other groups of these experiments. They follow, however, in size the lengths of the average normal curves. With the larger ones they are larger and with the smaller they are smaller. We obtain, then, a greater absolute difference sensibility in the case of the smaller ones than in the case of the larger ones. The average differences themselves are smaller with the larger curves than with the smaller curves. An undervaluation takes place with the former and an overvaluation with the latter. This overvaluation is greater than the undervaluation. From the standpoint of the average differences the difference sensibility is less in the case of the larger normal curves than is the

case of the smaller. The M. D. S. shows in the individual series that the individual differences where the average normal curves are large, decrease with the decrease in the individual normal curves, and where the average normal curves are small rather increase. This agrees with the results obtained from the average differences.

Group IV., where the metronome beat 120 times in the minute, shows in many ways results similar to those in group III. The average differences vary with one another somewhat more, and in series 2 the variation of the normal curves is greater than would be expected from group III. In series 3 it is smaller.

Group VI. was made so that the normal curves in series 1, 2 and 5 were made without rubbing the papers. The compared curves were described in the ordinary way, with the lead pencil. The M. R. V. shows that the increase of the individual normal curves in the case of the larger normal curves is accompanied by a decrease in their differences; in the case of the smaller by an increase. The variations of the normal curves show in series 5 a small variation, in series 1 and 2 a large one. From the standpoint of the variations the difference sensibility decreases with increase in the average normal curves. The average differences in series 1 and 2 give a mean which is smaller than the average difference in series 5. From the standpoint of the average distances, then, it is to be concluded that with increase in the average normal curves, the absolute and relative difference sensibility increases. This agrees with the results from the method of relative valuation. In general these series show no great deviation from group I. The rubbing in the case of the normal curves appears to exercise no great influence upon the valuation. Series 3 and 4 were made so that the arm was fatigued through lifting of a weight of five kg. It shows, however, nothing new.

Group V. was carried out from the standpoint of the variations of the compared curves. The normal curves were made in such a way that the eyes could see only and alone the making of the normal curves. By this it was thought to keep the pure stimulation or the normal curves more constant and to make the average differences follow more the real differences in the length of the motions or of the space valuations. The average normal curves varied in the 8 series between 6.17 cm. and 30.30 cm. Their variations give no regular increase or decrease with the same in the normal curves. The average of the larger average normal curves is greater than the average of the smaller. From this standpoint, then, the difference sensibility is greater in the case of the latter. It is, however, much smaller than in the other series. From 12.95 cm. to 15.79 cm. the average differences rise. From 20.65 cm. to 30.30 cm., and from 12.95 cm. to 6.17 cm., they decrease. In general the larger normal curves show much larger average differences than the smaller. The M. R. V. shows in general a small increase in the individual differences of each of the series with increase in the individual normal curves. In three series it is just the reverse. In the other five, however, it is so much to the contrary that an average increase takes place. The relative difference sensibility obtained from the average differences is greater in the case of the larger normal curves than in the case of the smaller. An exception to this occurs in series 6, where it is greatest. We find by the M. D. S. that the difference sensibility increases, in series 3, 4 and 7, with increase in the individual normal curves. In series 1, 2, 5, 6 and 8, however, it decreases.

Downwards movements. I consider now the downwards movements of observer S., by which changes in the difference sensibility caused

by changes in the lengths of the normal curves are sought. Only two of the twelve series had a more rapid rate of movement, namely, 120 beats of the metronome in the minute. In the case of the others it beat sixty times. The average normal curves varied between 2.14 cm. and 23.25 cm. Their variations increase gradually with increase in their lengths. From this standpoint, then, the absolute sensibility decreases gradually with the increase in the lengths of the average normal curves. The average differences, on the contrary, also gradually decrease with increase in the normal curves. On the whole they are very small. The greatest difference sensibility is at 11.50 cm. and at 4.40 cm., where the average differences are respectively —.05 cm. and +.05 cm. From these normal lengths it decreases with the longer and shorter normal curves. At 23-25 cm. the difference sensibility is greater than at 2.60 cm. (the average of series 5-9). Within the limits of the individual series the M. R. V. shows that a decrease in the lengths of the individual normal curves is accompanied by an increase in the size of the average differences, and this in proportion as the average normal curves increase. Series 11 and 12, which were made with the quicker rate of movement, show an increase in their average differences, and in their average normal variations above motions of the same size which were made with a slower rate. One series of the downwards movements, in which the normal curves were made without rubbing of the surface, gave an average normal length of 36.40 cm., with a variation of 4.35 cm., and an average difference of 12.77 cm. The contact of the hand with the table through the lead pencil appears, in the case of the downwards movements, to exercise a great influence. The M. R. V. shows an increase in the individual differences with decrease in the individual normal lengths.

Still another series was completed after fatiguing of the arm with a five kg. weight. The average normal length was 19.04 cm., their variation 2.47 cm., and the average difference 4.18 cm. The fatiguing brought out an overvaluation in comparison with series 3. In these downwards movements fatiguing causes the difference sensibility to decrease, both from the standpoint of the variation of the normal curves and from the standpoint of the average differences. The M. R. V. shows that an increase in the lengths of the average normal curves is accompanied more by a decrease in the average differences than by an increase.

The series so far described were made so that the starting points of the compared curves were the end points of the normal. Six series were now made in which the starting points for both normal and compared were the same. The lengths of the average curves varied between 8.12 cm. and 58.82. The variations and the normal curves themselves vary more in the case of the series considered above. They show no increase with the increase in the length of the average normal curves. The average differences are not very much different from the average differences of the other series. The greatest difference sensibility is at 17.90 cm. Over and under this it varies. It is least where the normal lengths are very great. The average differences vary between 0.00 cm. and —2.42 cm. In only one series is there an overvaluation. At 41.60 cm. the average difference is +1.90 cm., at 58.82 cm. it is 2.42 cm., and apart from these an undervaluation of from 0.00 cm. to 1.00 cm. is always present. The M. R. V., apart from series 1, where the average normal curve was 58.82 cm., shows an increase of the individual differences with increase in the individual normal curves.

Experiments with changes in the heights of the starting points. So far, we have considered these later results from the standpoint

of the lengths of the normal curves. As above mentioned, I have also carried out some such series to determine how far changes in the average differences were due to changes in the length of the normal curves. In the downwards direction thirteen series were made in two groups. In group I. the average normal curves varied between 13.75 cm. and 15.70 cm. Their starting points varied between 60 and 15 cm. under the horizontal plane. The average differences are for the lower of these, smaller, and for the higher, greater. The variations of the normal curves are greater in the first than in the last. The variations of the normal curves, then, show a smaller absolute difference sensibility at the lower points, while from the standpoint of the average differences a greater at these points. The M. D. S. shows more of a decrease of the difference sensibility with the increase in the lengths of the individual normal curves. The M. R. V. shows an approximate equal increase of the individual differences with increase in the lengths of the normal curves more and more as the starting points are raised.

Group II. consists of nine series, of which two are duplicates. The starting points vary between —45 cm. and +60. The variations of the normal curves all fall under 1.50 cm., and are somewhat greater above than below the horizontal point. The average normal lengths vary between 7.40 cm. and 12.95 cm. The larger of the same are the upper ones. The average differences are greater under than above the horizontal plane. In the former case there is an average overvaluation, in the latter an average undervaluation. None of the average differences amount to more than +1 cm. or to less than —1 cm. The absolute average undervaluation in the higher series is greater than the absolute average overvaluation in the lower ones. The M. R. V. shows in the former series an increase of the individual differences with the increase in the individual normal curves, while in the latter series a decrease is shown. In these latter series, we find that the increase in the average differences agrees with the increase in the individual differences, as shown in the separate series. The M. R. V. shows in the horizontal plane a decrease in the difference sensibility to be accompanied by an increase in the individual normal curves. With the heightening and lowering of the starting points this decrease becomes less, so that an increase takes place. On the average, also, there is an increase. Groups I. and II. are different from one another in so far as the first shows, on the whole, a larger overvaluation than the second.

Two groups of series of these kinds of movements were made in the downwards direction. In group I. the lengths of the normal curves varied between 8.22 cm. and 11.65 cm. Their variations are greater at the higher starting points, namely, 0.00, +15 and 30 cm. than at the lower, namely, —45, —30 and —15 cm., and also than at the highest, namely, +60 and +45 cm. The M. R. V. shows that the increase in the lengths of the individual normal curves at the lower points is accompanied by an increase in the lengths of the individual differences, but at the higher points by a decrease. The average differences varied between —1.00 cm. and +1.75. The average differences at points —45 cm., —30, —15 and at 0.00 are +1.17, —.55, —.87 and —.02 cm. At the points +15, +30 and +45, they increase. They are +1.36, +1.50 and +1.75 cm. At +60 it is —.9 cm. The variations of the lengths of the normal curves appear to have no influence on these average differences. We can assume, then, from these experiments that the difference sensibility is smaller above the horizontal plane than below it. Apart from the lowest point, —45, it is greater under. In group II. the normal

curves are longer. They vary between 16.72 cm. and 21.42 cm. The starting points were +15, +45 and +60. The variations are much the same as in group I. The average differences decrease with the increase in the lengths of the average normal curves and in the heights of the starting points. Along with this the difference sensibility decreases.

This finishes the results obtained from the one-handed experiments by means of the pencil method. The results are:— 1. The results obtained by the pencil method show that in valuing space distances, the angles at the shoulder joint should be taken as the measure of the sensibility in the making of arm movements.

2. The larger normal curves, when the movements are upwards, are more overvalued than are the smaller, where they are downwards more undervalued. This varies, however, with the observer and with the rate of movement.

3. The results obtained by the M. R. V. show that the individual compared curves increase on the whole with increase in their normal curves in both upwards and downwards movements. When the normal curves are small they rather decrease, however. This relation between the normal and compared curves is dependent also on the length of the normal curves, as also on the heights of the starting points.

4. The variations of the normal curves show that the absolute difference sensibility is greater where the normal curves are smaller than where they are larger, because the variations are greater in the latter case than in the former.

5. The upwards movements at points above the horizontal plane are overvalued, and under it they are undervalued. The reverse holds where the movements are downwards. This over and undervaluation varies more in the former than in the latter movements. Observer C., who was seated during the experiments, overvalued upwards movements above the horizontal plane and undervalued downwards movements under the same plane.

6. Very rapid rates of movement caused greater average differences in the larger and smaller normal curves than medium rates of movement.

7. It is impossible to overcome great differences in the valuations which depend upon the individual observers, because the shoulder joints in the holding of the arm out and moving of it allow the curves to vary too much on account of the radii and centres of movement.

B.

The Two-Handed Movements.

a. Influence of the Starting Points.

We consider, first, the differences in the individual series, which are due to changes in the heights of the starting points, in the lengths of the normal curves. In such psychological curves as these, where one investigates the ability to make two movements equal to one another, we have to deal with two groups of sensations rather than with two single sensations, as in the case of tone investigations, for example. For we can say that the two motions are equal to one

another when the two groups of sensations are equal. I give, first, the results of the changes in the heights of the starting points in influencing the valuation of these two groups. The heights of these remain for the normal curves in the same series always the same, while those of compared curves change with each series. In the upwards direction five groups were made, two by observer Kp., one with the right hand for the normal movements and one with the left hand for the same; two by O. (observer) S., and one by O. K. Each group had six or eight series and each of these is the average of ten experiments. In the diagrams 1, 2, 3, 4, both the upwards and downwards groups of movements are described.[1] Loeb (l. c.) has shown by his method that in upwards movements, where normal and compared movements are made simultaneously (simultaneous movements), the compared decrease more and more in comparison to the normal. They are undervalued. In the five groups considered now apart from changes in the lengths of the normal curves, there is only partly such a decrease. In groups c and h of O. S., such a decrease is most distinctly present. In group h the decrease from starting point (S. P.) —4 cm. to +6 is somewhat more than 2 cm. From S. P. +6 cm. up to S. P. +16, it is somewhat more than 1 cm. From starting point +16 up to +26 it increases a little, and then from +26 to +36 decreases nearly 2 cm. It is to be said here that in all these two-handed experiments the starting points of the normal hands were taken for observer Kp. at —40, for K. at —6, for R. at —10, and for S. at —4. In group c of the last there is an irregular increase from S. P. —24 up to S. P. +26, where a small increase at point +26, and then again a decrease up to +55, takes place. In the two groups, a and g, of O. Kp., the decrease is only partly to be met with. In group a, the line (Diag. 2)[1] sinks from S. P. —40, nearly 2 cm., down to —30. From —30 to —20 it rises 3.5 cm., and from —20 to —10, 1.50 cm. From —10 to 0.00 it decreases 3 cm., and at +10, .2 more. From +10 up to +20 a great deviation takes place and then it rises 2 cm. up to +30 cm. In this group, then, the increase and decrease in the differences vary. At first there is a decrease, then an increase, again a decrease and then again an increase. This variation is not dependent on the lengths of the normal curves, for the average of these does not vary sufficiently. Group b, of O. R. (Diag. 2)[1], shows an increase in the first three series and a decrease in the last three. This variation, with the increase in the heights of the starting points, agrees with group a of O. Kp., but agrees only partly with Loeb's results. In the other group, g, of O. Kp., there is an alteration between an increase and decrease, and *vice versa*, with each new starting point, and the results at the last starting point do not vary much from those at the first. The large variation in the lengths of the normal curves makes it possible that these last served as a means in making the valuations correct. We will later become acquainted with the influence which the lengths of the normal curves exert on the average differences. On the whole it is to be said that with upwards movements there is only partly an increase in the lengths of the compared curves in comparison with those of the normal where the starting points rise. It is worthy of remark that in group g, a, b and c, an inversely correct valuation takes place at the horizontal plane. From this point the average differences increase and decrease. Six groups of downwards motions were carried out, two by O. K., two by O. R. and two by O. S. In each case one was made with the right hand, for the normal curves, and one

[1] See original.

with the left for the same. In group d, of O. K., an increase in the average differences occurs from S. P. —6 up to +34. From here up to +64, a decrease takes place. Inversely, in group l of O. K., there is a decrease from —6 to +14 and an increase from +14 to +54. Loeb found that by his method an overvaluation took place with increase in heights of the starting points where the motion was downwards. He says, however, that these movements are not so correctly made as those in the opposite directions. In this group l of O. K., it is possible that the difference in the lengths of the normal curves from S. P. —6 to +14, makes a sufficient difference in the compared curves to explain the decrease. In group d there is also a difference in the normal curves in connection with the decrease in the starting points. I think, however, that this is more dependent on the heights of the starting points than on the differences in the lengths of the normal curves. We will later take the opportunity of treating this further. Group e of O. R. shows a regular increase from starting point —10 up to +30, which is possibly dependent upon changes in the lengths of the normal curves. Group m, by the same, shows five changes between increase and decrease from points —10 cm. up to +50. From +10 to +30 we find, with the exception of a decrease from 1.50 cm., between +10 and +20, a rather regular increase. From +30 to +40 it is possible that the difference between the lengths of the normal curves causes the decrease. There remain two groups of O. S. In group f we meet with a regular increase from —4 up to +26. From +26 up to +46 a decrease takes place, which is accompanied by a shortening of the normal curves. Between +46 and +56 an increase again is present. In group n a decrease, at first from —4 to +6, is seen, and after this a distinct increase from +6 up to +16. Between +16 and +26 a decrease occurs, and from +26 up to +56 the increase is clear enough. Probably the difference, 3.50 cm., in the lengths of the normal curves accounts somewhat for the small increase between points +26 and +36.

Apart, then, from the consideration of the differences caused by changes in the lengths and variations in the normal curves and by the time relations of the same, we can only agree partly with Dr. Loeb's results. Movements in both directions show variations from the increase and decrease in the differences which his method gives. These are greatest where the starting points for the compared movements are taken very high or very low. They also depend upon the observers.

b. Influences of the Lengths of the Normal Curves.

I give now some remarks upon the differences which are caused by the changes in the lengths of the normal curves. I treat the series each by itself, because otherwise the heights of the starting points would exercise too much influence. The individual series show the effect of the changes in the lengths of the normal curves without the variations which arise through these latter. Preliminarily we can say that changes in the lengths of the normal curves do influence the compared ones.

To show this I have used the M. R. V. The average of all the values obtained by this method for the upwards movements was 41.32, for the downwards 41.8. In the former, therefore, the individual compared curves decrease in comparison with their normal ones, with increase inthese latter. In the latter they rather increase than decrease. This increase is, however, rather small. One might say that in this case the normal and compared curves increase and

decrease relatively to one another, with increase in the lengths of the normal curves. This signifies that if a normal curve has a certain value, each increase of 1 cm. in the normal curves is accompanied by an increase of a little more than 1 cm. in the corresponding compared curves where the movements are downwards. Where these are upwards, the accompanying increase will be a little less than 1 cm. These increases and decreases, as determined by the method of relative valuation, vary according to the observer and the heights of the starting points. In group m of O. R., for example, this decrease rather increases with the elevating of the starting points. The average normal curves themselves do not vary much. In group e the average of the values shows an average increase, while in group m an average decrease. This agrees with the average differences. Again, in group l of O. K., a decrease obtained by this method shows itself from points —6 to +14, and from +14 to +54 an increase. This agrees with the decrease in the average differences from —6 to +14, and with the increase from +14 to +54. At starting point +14 the very high number, 96, is worthy of notice. In group d the values obtained by the M. R. V., from points +46 on, increase and decrease with the same of the average differences. Group n of O. S. shows at S. P. +36, and at +46, higher values, which accompany the increase in the average differences, and group f a high value, 80, corresponding to the greater increase in the average differences. The upwards movements of group g of O. Kp., apart from S. P. 0.00, show an agreement between the alterations in the average differences and in the values obtained by the M. R. V., so that an increase in the one accompanies that in another; so also with a decrease. From starting points —40 to —10, group a of O. Kp. agrees with the above, as also from S. P. 0.00 to +40. In group b of O. R. the increase and decrease obtained by this method follow the same in the average differences. In group h the values obtained by this method appear to show the opposite results to that of the increase and decrease in the average differences. From points +6 to +36 these values do not vary much, however, from such as are given by the differences when a relatively correct valuation takes place. In group c the values obtained by this method agree with the increase accompanying the elevating of the starting points of the average compared curves, in so far as both show an increase from —24 to —14 and under, from 0.00 to —56 a decrease. If we put these results now together, we find that the decrease and increase of the compared curves in each series, in comparison with their normal ones, agree sufficiently nearly with the decrease and increase in the average differences. From this it is to be assumed that the compared curves, in comparison with their normal ones in the separate series, vary according to the heights of the starting points and the direction of the motions. This presupposes the same for the differences between the average differences. The values obtained by the M. R. V. show in the downwards movements more of an increase than a decrease of the individual compared curves in comparison with their normal curves. This agrees with the average increase of the average compared curves over the average normal curves. On the contrary, a similar decrease is shown by the corresponding values of the upwards movements, and this agrees with the average decrease of the average compared curves over their normal curves. These values vary, also, as we have shown above, according to the heights of their starting points.

In both cases we find, however, that this decrease obtained by the M. R. V., in the case of the upwards movements and increase

in the case of the downwards, is not altogether complete. Were this so, then the values attained by this method for the decrease would be 0.00, and for the increase, 100. They are only relative values. In both cases the average values show deviations from the above. It is only the variations of these values which are large, according to the differences in the starting points, lengths of the normal curves and direction of the movements. It will still be our duty now to again consider the individual series and groups of series of the different observers in so far as all three changes mentioned above work together.

The Combined Effect of Changes in Heights of the Starting Points in the Lengths of the Normal Curves and in the Direction of Movement.

The average differences show in general, with few exceptions, that at and above the horizontal point, the upwards movements exhibit a decrease of the average compared curves in comparison with their normal ones with the raising of the starting points, while the downwards ones show an increase. This is not very continuous in these experiments on account of the variations in the average normal curves and in the observers. After this general conclusion in regard to differences due to changes in the direction of movement, I consider now the combined effect of changes in the lengths of the normal curves and in the heights of the starting points.

(*a.*) In the downwards movements. The two groups of experiments of O. K. show that where the right hand makes the normal curve and the heights of the starting points are gradually increased from —6 to +34, and the left hand the normal curve with increase in the points from +14 to +54, the average compared curves in comparison with their normal curves more and more increase. In these groups the variations in the average normal lengths are small enough to leave them out of account. In group e of O. R., on the contrary, one can only compare the values from points +20 to +60 with one another. The variations of the average normal curves within these limits are not great. They show a distinct increase in the average differences with the increase in the average normal curves. In group m of O. R. the decrease in the average differences accompanying the raising of the points is dependent on this changing of the starting points. This occurs very exactly between points 0.00 and +30, and the variations in the average normal lengths between these points are very small. In the right handed group of O. S. there is a distinct increase in the average differences between the points —4 and +26, dependent on the changing between these points. From +26 to +46, the normal lengths vary too much to compare them with one another, while from points +46 to +56, there is again an increase in the average differences. In the left hand movements (where the left hand makes the normal curve) of O. S. there can only be considered an increase from points +36 up to +56. From the points +6 to +26 there is only a very small increase, and the variation of the average normal lengths between these points and between +36 and +56 is too great in order to compare the average differences between +6 and +56 with one another. In the downwards movements, then, apart from the left hand group of O. R., we find an increase in the average differences with the increase in the average normal lengths, where the variations in the average normal lengths in the groups are not large.

(*b.*) In the upwards movements. In group a of O. Kp. there is a decrease in the average differences between points —10 and +16 distinctly due to the changes in the starting points, because the

average normal lengths vary between these points very little. On the contrary between points —30 and —10 there is an increase. Between these points also, the average normal lengths vary very little. From these groups it is to be assumed that lowering of the starting points under the horizontal plane makes the average differences increase, but above the same, decrease. In group g of O. K. there is under the horizontal plane, —40 and —30, an increase, but above the same, between +10 and +20, a decrease. Apart from these groups the variations in the average normal curves are too great in order to compare the average normal differences with one another. In group h of O. S. there occurs between points +6 and +26 a distinct decrease in the average differences, which is dependent upon the raising of the starting points, because the variation of the average normal lengths is not great. Apart from these, there is in general a decrease, which, however, can be ascribed to the variation in the average normal lengths and not to that of the starting points. Group e of O. S. is in every way similar to group h. Group b of O. R. shows the average differences to decrease between +10 and +20 and between +30 and +40. Otherwise the variations in the average normal curves are too great in order to determine the influence of the starting points. In the upwards movements we have found that the average differences decrease with raising of the starting points. It would be possible, where this variation in the average normal lengths, above spoken of, is large in the groups, to reckon out a curve which would show the influence of the changes in the lengths of the average normal curves. In so far, however, as we have already found that, where these variations are not too great, in the upwards movements a decrease, of the compared curves in comparison with the normal ones, in the downwards ones, on the other hand, an increase, occurs, there is no purpose to be fulfilled. My experiments show, nevertheless, that the size of this increase and decrease is very much dependent on the observer. This can arise from the fact that the starting points of the normal curves bore different relations to the horizontal plane for the different observers. The normal starting points for observers Kp., K., R. and S. were respectively —40, —6, —10 and —4.

c. What kind of symmetrical results are given? There occurs a general tendency for the different compared curves to be made greater in comparison with their normal curves where the right hand is the normal than where the left hand is the same. This tendency varies with the observer and also with the height of the movement. The average of the average differences of the different series is reckoned out for each group. This average of a left hand group minus that of a right hand group gives the individual differences, which show the tendency above described. In groups d and l of observer K. this is very distinct. Here the left hand average is —3.05 cm., that of the right +3.23. The difference is 6.28 cm.; also in groups c and m of observer R. and in groups d and l of observer S., this tendency is clear. In the former two the average of the left hand motions is —.50 cm. and that of the right hand +3.93 cm., a difference of 4.13 cm. In the last two series the average of the left hand motions is —.19, that of the right hand +4.11, a difference of 4.30 cm. In the upwards movements in groups a and g of observer Kp., this tendency is not so distinct. The average of the left hand movements is —2.63, that of the right hand —1.21 cm., a difference of 1.42. In general, however, it is present especially at the beginning and end of the group. It is possible that the large variations in the lengths of the normal curves will explain this somewhat. In group a the limits between the largest and smallest

normal curves are roughly 13 and 22 cm. In group g they are 22 and 37 cm. In groups c and h of observer S. the general tendency is also present. The average of the average differences of the left hand movements is —1.17, that of the right +0.515 cm., a difference of 1.68 cm. In group b of observer R., we find no variation from the tendency in so far as the compared curves are less in extent than the same in the downwards group, which were right handed. The average of the former is +2.39 cm., that of group b +3.93 cm., and the difference is 1.54 cm. We have no corresponding group of the left hand upwards movements in order to compare it with group b.

The above mentioned figures of the M. R. V. show that from the standpoint of the relation of an increase in the individual normal curves, when one considers the series separately, to the making of their compared curves, a distinct asymmetrical phenomenon is present. Where the left hand makes the normal movements the compared curves decrease with increase in their normal curves more than where the right hand makes them. The M. R. V. yields values for the upwards movements of O. Kp., where they are left handed, of 36.5, and where they are right handed, 38.4. For O. S. the corresponding values are 38.4 and 45. Where the movements are in the downwards direction, the corresponding values for O. K. are 55 and 50. For O. R. the corresponding values are 42.7 and 59.5, and for O. S. 47.5 and 56.3. The single exception to this asymmetrical difference consists in O. K.'s case, where it is of the opposite nature.

Another asymmetrical difference is found in the averages of the average normal curves for the right and left hand groups along with the variations of these. While there was no strictness as to the making of the average normal curves in the series for the groups equal for the right and left handed experiments, still the variations in these show certain degrees of difference sensibility. The averages of the average normal curves in the downwards movements are for the observers R. K. and S., for the left hand, 15.00 cm., 27.17 and 12.42, and for the right hand, 11.14, 21.23 and 11.14 cm. respectively. It results, then, that the average of the average normal curves of each left hand group is greater than the same of each corresponding right hand one. Further, the averages of the average differences as we have mentioned above are similar to these. With each observer the average of the average differences of all the series of a right hand group is 4 to 6 cm. smaller than the same in a right hand group. This is so significant that an average undervaluation takes place where the groups are left handed, while an average overvaluation takes places in the right handed ones. The M. R. V. has shown that the compared curves in comparison with their normal curves decrease with increase in these last.

These three asymmetrical tendencies work together. The variations of the normal curves when a whole group comes into consideration give the decrease obtained by the M. R. V. an importance. If, *e. g.*, each increase in the normal curves of 1 cm. produced a difference of only +.04 cm. or +.05 in the difference between a normal and a compared curve, this could produce in 75 to 100 such, as occurs in a single group, a difference of 2.00 cm. to 3.00 cm. In some of the groups the average of the variations is 2 cm., and in all it is not less than 1.00. The differences between the limits of the largest and smallest normal curves of the groups themselves lie between 2.53 and 11.00 cm. There is here plenty of room for the decrease, as shown by the M. R. V. This decrease explains to some extent the undervaluation in the left hand groups and the overvaluation in the right hand. In the two downwards

groups of O. R. the averages decrease with raising of the starting points in the left hand movements. In right hand ones they increase. It can then scarcely be doubted that this difference between the average differences in the left and right hand groups is dependent on the corresponding results obtained by the decrease and increase in the values obtained by the M. R. V. With the other observers this difference is not so easy to explain in this way. We have referred to this above when we considered the influence of the changes in the starting points and in the normal curves upon the difference sensibility. It results, then, that in the downwards motions these three asymmetrical phenomena, namely, the average overvaluation of the right hand groups and undervaluation of the left hand, the greater extent of the average normal curves of the left hand than that of those of right hand, and the smaller values obtained by the M. R. V. in the left hand movements than in the right hand ones, accompany one another. Where the movements are in the upwards direction, the average of the average normal curves of the left hand group of O. Kp. is smaller than the same average of the right hand group, while by O. S. it is inverse. Here the average of the average normal curves of the left hand group is greater than the same of the right hand group, as we have found above with the downwards movements. In these upwards movements the difference between the average differences of the right and left hand groups is not at all so great as in the downwards ones. M. R. V. shows that a much greater decrease in the individual compared curves in comparison with their normal ones takes place in the right handed upwards movements than in the left handed.

d. The variations. I have so far considered the simple average difference without respect to the average variation, because the variations of the normal curves produce an effect upon the average variations of the different series of the groups. The average of all the average variations of the normal curves of the different series of O. Kp. is 1.991 cm. and of the compared curves 1.643 cm. The difference between the two is .348 cm. In the series of all the other observers the corresponding averages are smaller for the normal curves than for the compared. We call O. Kp.'s difference +.348 cm. The corresponding difference with O. K , is —.510 cm. The average of all the variations of his normal curves is 1.47 cm. and of his compared ones 1.98 cm. The corresponding averages of O. R. are 1.263 cm., and 1.47 producing a difference of —.206 cm. The corresponding averages of O. S. are 1.08 cm. and 1.25 cm. and the difference is —.17 cm. It appears now that the compared curves do not vary very much from the normal. What effect now do these variations of the normal and compared curves produce on the average variations of the series? Or of what importance are the latter? The average variation spoken of here was reckoned out as follows: The individual difference of the ten experiments give us +, — and 0 values. All the plus subtracted from all the minus give us an average difference, which can be +, — or 0. This average difference is now subtracted from the individual differences. The values so obtained are half plus and half minus when added together, and the average obtained by dividing by ten is the average variation of the series. The average of all these for O. Kp. is 1.84 cm.; for O. K., 2.04 cm.; for O. R., 1.363 cm. and for O. S., 1.38 cm. The average of the normal curves lies between 10 and 30 cm. In any case, then, these average variations are not very large. I have not reckoned out the differences between the average variations and the average differences in order to obtain from this the raw values for the sen-

sibility, because these variations are possibly too much influenced through the variations in the normal curves. Still I think that by further investigation these variations in connection with the average differences would give better values for the sensibility than the simple average differences. From the investigation, we have so far obtained no exact sensibility values for the different starting points. I think that the sensibility is very variable with the different observers,[1] so that one cannot obtain exact values for the difference sensibility, as has already been done in the other fields of psychology.

e. Influence of the metronome. In all these experiments the metronome beat sixty times in the minute. The duration of each motion could, then, equal one second. Where the two motions of each experiment were made together at the same time, they were made with each beat of the metronome. In case the compared motion followed the normal one, they were made so that the former followed the latter on the next beat, by O. K. and O. S., or the fourth beat, as by O. Kp. I find the great differences between the results of the variations of the normal and compared curves, which exist in the figures of O. Kp. and the other observers, to arise in this difference in the beating of the metronome. As above shown the average of all the variations of the normal curves is greater than the same of the compared ones by +.34 cm. In the results of the other observers it is smaller by +.29 cm. If the sensibility for space valuations from the standpoint of the different variations is very good, then the variations of the normal and compared curves would be equal. If the movements are made at the same time or immediately after one another, the variations of the normal and compared curves do not change. If the latter, however, accompany the fourth beat of the metronome, then their variations will be smaller because they are more influenced by the metronome, and the normal curves would also serve to give a better sensibility. The above described figures show this.

f. Influence of the weights. I have made forty-one series of experiments in order to estimate the influence of the weights upon the valuations of the motions. Twenty-three were made by O. S., twelve by O. R. and six by O. Kp. In the first twenty-three a weight of 1.31 kg. was fastened to the hand, which made the compared motions, and the starting points were varied, as in the above described experiments. Four groups of these are described on the diagrams X, R, j and K. The weight remained constant in these. In group x there is no great deviation from the ordinary group c as the normal lengths increase. The great deviation in group R is dependent on the time relations. In group R the compared motions were made at the same time as the normal ones. In group x they followed them. Group K agrees in general with group h. The curve for k stands only a little lower. Where the motions were downwards, group j gives a curve higher than group n. The results of groups R and j in comparison with group c agree with the results, which show the differences in the lengths of the normal curves. The groups 1, 2, 3, 4, 5 and 6, which are described to the left on diagram 2, were made by O. Kp. and O. R. with the left hand as the normal, and with weights of 0.5 kg., 1.31 kg. and 3 kgs. They were in the upwards direction. It is worthy of remark that all these groups of O. Kp. appear higher on the diagrams than his group a, which is to be taken into comparison with it. In groups 1 and 2 one observes an increase in the compared curves with the increase in the weight.

[1] "Der Muskelsinn der Blinder." Paul Hocheisen, Berlin.

In group 3 this is not continued. Groups 4, 5 and 6 of O. R. also show an increase in the lengths of the compared curves with the increase in the size of the weights, if one takes into consideration the increase in the lengths of the normal curves. These 3 groups appear lower (on the diagram) than group b of O. R. The groups 7, 8 and 9, diagram 3 of the downwards movements of O. R., with starting points of —2 cm., show an unsteady increase of the average differences, if the increase in the averages of the normal curves is considered. This increase is more doubtful in the groups where the points are —10 cm. In groups x and R of O. S., in groups j, 7, 8 and 9 of O. S. and O. R., respectively, and in groups 1, 2, 3, 4, 5 and 6 of O. Kp. and O. R., respectively, the average differences are greater where the motions were successive than those of the simultaneously made movements.

Results of the Pencil Method, Two-Hand Motions.

1. The results obtained by the M. R. V. show that where the normal and compared motions are considered in each series separately, an increase in the former is accompanied by a comparative decrease in the latter if the direction is upwards, if it is downwards by an increase. In both cases the starting points are above the horizontal plane, and the above results are more distinct the higher these points are raised.
2. With the raising of the starting points of the compared curves above the horizontal plane, the average differences increase in downwards movements, and decrease in upwards according to the observer and heights of the normal curves.
3. The unsteadiness as shown by comparing the average normal curves with one another by the different observers; makes it impossible to obtain the proper values for estimating the absolute or relative difference sensibility of these two-handed movements.
4. The movements where the right hand is the normal, exhibit an average overvaluation, and inversely where the left hand is the normal.
5. Where the left hand is the normal the average normal curves with equal effort of the will are greater than where the right hand is the normal.
6. The values obtained by the M. R. V. are, for left handed movements, less than those for right-handed.

The experiments with the weights show:—

7. Firstly, the average differences where a weight is used are larger than where the movements are made without such, in the case of downwards motions, and in the case of upwards motions smaller, apart from groups 1, 2 and 3 of O. K.
8. Secondly, the average differences of the successive movements are larger than those of the simultaneous.
9. Thirdly, the greater the weights are, the more the average differences decrease in upwards movements, and in downwards the reverse.

These experiments with the weights are, however, not sufficiently numerous, for the different observers, in order to lay much value on them.

II.

Experiments by the Angle Method, Using the Method of Just Perceptible Change.

General Description.

These experiments were made with Dr. Külpe, K.; Mr. Child, C.; Dr. Cohn, Ch.; Mr. Rogers, R., and the investigator himself, S., as observers. Eight series were obtained from O. K., four from O. C.,

four from O. Ch., two from O. R., and one from S. Some of the experiments of O. R. were intended to determine differences between valuations of active and passive movements. Those of the other observers were active motions with the horizontal plane as starting points as well as end points with motions whose starting points were above and below this. The rates were fixed by the beating of a metronome forty and one hundred and twenty times in the minute. The results obtained, where observer K. made the valuations, were in all the above directions and at all the different rates. The diagram[1] given shows the investigated extents of movement and the directions of the same. A is the centre of motion for the arm, BA is the horizontal plane, in which B is the starting point for downwards movements under this plane, and for upwards above it, and inversely the end point for upwards movements under, and for downwards movements above this. Downwards movements were made under and above this plane with normal distances of 15°, 30° and 60°, and with both rates. A few series of these motions were also made with 10° as normal extent under the 0 point (horizontal plane). The experiments of O. C. include the above described motions which lie under the 0 point, those of O. Ch. such as lie above it. I consider the experiments in the following order: those of each observer separately, those under the 0 point and afterwards those above it; at first the downwards experiments under the 0 point, then the upwards; the upwards above the same next, and lastly the downwards above it.

The two chief rates yield eight groups[1] of series for O. K., four for O. C. and four for O. Ch. In the tables the symbols[2] represent the following values: "r" is equal to the normal stimulus, r_o is the mean of the just greater than and the valuation of the extent above the same and just equal to it. "r"$_u$ is equal to the mean of the stimulus just smaller than the normal, and the valuation of the extent below the normal just equal to it. R is the mean of r_o and r_u. $\triangle r_o$ is equal to r_o-r, $\triangle r_u$ is equal to r-r_u. $\triangle r$ is equal to the mean of $\triangle r_o$ and $\triangle r_u$, while $\triangle$ is equal to R-r. Vr_o is the variation of the parts of r_o, and Vr_u that of those of r_u. Zr_o is the number of the parts of r_o, Zr of those of r_u. R shows the relative over and undervaluation. $\frac{\Delta r}{r}$ shows the approximate relative sensibility.

a. Under the Horizontal Plane.

Observer K. Group I. of O. K. consists of four series of downwards motions of extents of 10°, 15° and 30° under the horizontal plane, with a rate of forty beats of the metronome in the minute, and of one series of 60° at a rate of 120 beats a minute. We consider the first four series first. The variations in these vary between .37° and 1.00°. At 10° and 60° they are greater, while at 15° and 30° they are nearly alike. The valuating is, then, steadier at 15° and 30° than at 10° and 60°. The thresholds for difference sensibility, as shown by the values for $\triangle r$, which are .42°, .70° and .59°, increase from 15° to 30° and then decrease. This shows that the absolute difference sensibility does not increase regularly. The relative difference sensibility increases with the increase in the normal lengths, more between 10° and 15° and 30° and 60° than between 15° and 30°. At 15°, 30° and 60° the valuation is nearly correct. At 10° it is +.50°; at 15°, +.04°; and at

[1] See original.
[2] See Wundt's *Gründz. d. Phys. Psych.*

30° and 60°, —.05 and —.03°. The variations of all the series considered here of 10° normal extent can only be explained by the fact that the changes in the gradations of the compared extents was 1° instead of ½°. Still another series of 60° normal extent at the rate of twenty beats a minute shows an approximately correct valuation, namely, an undervaluation of .09°. The variations are greater than those at the more rapid rate. The threshold is .40°. The difference sensibility obtained by $\frac{\Delta r}{r}$ is finer than that of those of 60° at the more rapid rate.

Group II., of observer K., shows valuations of downwards movements at 10°, 15°, 30° and 60° under the horizontal plane at a rate of 120 beats in the minute. The variations of these vary for 10° and 30° between .68° and 1.25°, while for 15° and 60° they are respectively .30° and .63°. The individual valuations of these series are not so numerous as are those of the first. At 10° and 60° there is a large undervaluation, respectively .62° and .81°. At 15° one finds an undervaluation, of .12°, and at 30° an overvaluation of .19°. The thresholds, and along with them the absolute difference sensibility, increase with the increase in the normal extent. They are respectively for 15°, 30° and 60°, .41°, 1.05° and 1.31°. For 10° it is .87°. The relative difference sensibility is finest for 60°, less at 15°, still less at 30°, and still less again at 10°. From the standpoint of both thresholds and over and undervaluations, as also of the relative difference sensibility, the slower rate of motion is best in making downwards movements.

Group III. of O. K. was made in the upwards direction towards the horizontal plane at angles of 15°, 30° and 60°, and at rates of sixty beats of the metronome in the minute. The variations are, at 30° and 60°, more equal to one another than at 15°. Therefore, the steadiness is greater at 10° than at 30° and 60°. For angles of 10° and 60° overvaluations take place, and for 30° there is an undervaluation. The thresholds for 15°, 30° and 60° are respectively .34°, .48° and .65°. These signify an increase in the absolute difference sensibility with increase in the normal extents. The relative difference sensibility increases with the increase in the lengths of the normal angles (or curves, as they are seen on the table). The very small thresholds of this group are worthy of remark.

Group IV. of O. K. consists of upwards movements towards the horizontal plane at angles of 15°, 30° and 60°, and at a rate of 120 beats of the metronome in the minute. On the whole, undervaluations are manifest. These valuations at 15° and 60°, taking that at 30° as a standard, are the inverse of those in group III. In the last the angles at 30° are more overvalued than at 30° and 60°, while in group IV. the reverse is true. The beating of the metronome is the cause of this, which we shall consider later. At 30° the valuation is nearly correct, namely, —.07°. At 15° and 60° they are —.08° and —.13°. The thresholds are for 15°, 30° and 60°, respectively, .94°, .72° and 1.25°. They are also much greater than at rates of forty beats in the minute. Hence the absolute difference sensibilty increases regularly with the increase in the normal angles. On the other hand, the relative difference sensibility also increases with the same increase. The variation is at 30° smallest, at 60° larger, and at 15° greatest. They vary between 0.00° and .78°. We find now that the time relations have more influence upon the valuations than the directions. In the case of O. C. we will find that the direction exercises the greater influences.

Observer C. Group I. consists of downwards motions at 10°, 15°, 30° and 60° under the horizontal plane at a rate of forty beats of

the metronome in the minute. Vr_o and Vr_u vary between .50° and .90°. At 10° and 30° they are smallest, and at 15° and 60° largest. From the standpoint of the variations, it is to be assumed that the difference sensibility is smaller at 15° and 60° than at 10° and 30°. The threshold values ($\triangle r$) are for 10°, 15°, 30° and 60°, respectively, .87°, .76°, .87° and 1.16°. From 15° to 60°, then, the absolute difference sensibility decreases. At 10° it is the same as at 30°. R and $\triangle$ show an approximately equal valuation from the standpoint of the average valuation. At 10° there is a correct valuation. From 15° on the small undervaluation decreases up to 60°, where there is an overvaluation. $\frac{\Delta r}{r}$ shows the relative difference sensibility with increase in the normal angles to increase distinctly.

Group II. consists of downwards movements at 10°, 15°, 30° and 60° under the horizontal plane at a rate of 120 beats of the metronome in the minute. The variations in the values are smallest, namely, .50° and .75° at 10° and 30°, and largest at 15° and 60°, namely, 1.03° and 1.15°. At 10° there is a combined overvaluation of .25°, which increases to .50° at 15°, and at 30° again to .315°. The thresholds of 10° and 30° agree again with one another in this group. They are smaller than at 15° and 60°. At the first they are .75° and 1.09°, and at the last 1.25° and 1.37°. The absolute difference sensibility is, then, smaller at the first than at the last. $\frac{\Delta r}{r}$ shows that the relative difference sensibility decreases from 10° to 15° and increases from 15° to 60°. The difference between the relative sensibilities at 15° and 30° is greater than that between 30° and 60°. It results, then, that a slow rate of movement is better in valuing downwards movements than a more rapid rate.

Group III. consists of downwards movements at angles of 15°, 30° and 60° below, but towards the horizontal plane, at a rate of forty beats of the metronome in the minute. The number of individual valuations is, in the experiments that follow, less than formerly. At 15° there is an overvaluation of .26°, at 30° and 60° undervaluations of .21° and .13°. The thresholds increase gradually from 15° on. They are respectively .92°, 1.29° and 1.94° somewhat greater than in the downwards movements. The absolute difference sensibility decreases, then, with the increase in the normal extents. $\frac{\Delta r}{r}$ shows that the relative difference sensibility rather decreases between 15° and 30° and increases between 30° and 60°. The variations remain more constant in these groups than in the downwards movements.

Group IV. consists of downwards movements at angles of 15°, 30° and 60°, as in group III., at a rate of 120 beats in the minute. At 15° and 60° there are undervaluations of respectively .32° and .04°. At 30° an overvaluation of .34° is shown. The variations are also greater at 15° and 60° than at 30°. On this account there is a greater absolute difference sensibility at 30° than at 15° and 60°, from the standpoint of the variations. The thresholds increase in size from 15° on. They are .62°, 1.34° and 2.34° respectively for 15°, 30° and 60°. Groups III. and IV. show that the downwards movements are made more exactly from the standpoint of these than are the upwards. The relative difference sensibility obtained from $\frac{\Delta r}{r}$ does not agree with that obtained from the thresholds. At 60° it is finest, at 15° finer, and at 30° and 60° it is smallest.

One series was also made in the downwards direction by O. R. at 15° and at a rate of forty beats of the metronome in the minute. The valuation was very nearly correct, namely, an overvaluation of .04°. The accompanying variations were also small, .16° and .22°. The threshold agreed with that of the upwards movements. It was 1.52°. The small variations of the series of O. R. show that they are very valuable, although the thresholds are large.

I add here a series by O. S., which was made in the downwards direction below the horizontal point at angles of 30° and a rate of forty beats of the metronome a minute. An overvaluation of .37° took place, while the variations were .75° and .74°. The threshold and the relative difference sensibility agree with the experiments of O. C. and O. K. There is more of an overvaluation in these experiments than in the others.

b. Above the Horizontal Plane.

Observer K. The groups V., VI., VII. and VIII. of O. K. follow. They were made in the upwards and downwards directions above the horizontal plane at angles of 15°, 30° and 60° and at rates of forty and one hundred and twenty beats of the metronome in the minute.

Group V. consists of upwards movements at angles of 15°, 30° and 60° at rates of forty beats in the minute. The variations increase with the increase in the normal angles. △ shows at 15° an overvaluation of .12° at 30°, an undervaluation of 16°, and at 60° one of .26°. The thresholds gradually increase with the increase in the normal angles. They are .37°, .59° and 1.12°. Hence the absolute difference sensibility decreases with the increase in the normal angle. $\frac{\Delta r}{r}$ shows a greater increase of the relative difference sensibility between 15° and 30° than between 30° and 60°.

Group VI. was similarly made to group V., apart from its more rapid rate of movement, which was 120 beats instead of forty in the minute. The variations increase with the increase in the normal angles. The valuations are the reverse of those in group V., in so far as the compared movements in comparison with the normal ones increase with the increase in these last. There is an undervaluation of .07° at 15°, one of .15° at 30° and of .56° at 60°. At 30° it is smaller than at 15° and 60°. The thresholds are: .56°, .40° and 1.19°, respectively, at 15°, 30° and 60°. At 30° it is somewhat smaller than at 15° and 60°. The absolute difference sensibility is smaller at the very large angles, greater at the medium ones and somewhat less at 60° than at 30°. The above mentioned differences between the valuation in series V. and VI. can only be explained through the change in the rate of movement.

Group VII. of O. K. consists of downwards movements from above towards the horizontal plane at angles of 15°, 30° and 60° and at rates of forty beats of the metronome in the minute. The variations increase here also with the increase in the normal angles. They vary between .16° and .75°. The valuations are related to one another similarly as in group V. At 15° there is an overvaluation valuation. The compared movements decrease, then, in comparison of .05° cm., at 30° an undervaluation of .10° and at 60° a correct with their normal ones with increase in these last. The thresholds are: .37°, .53° and .75° for 15°, 30° and 60° respectively. Hence the absolute difference sensibility decreases with increase in the normal angles. The relative difference sensibility increases more between 15° and 30° than between 30° and 60°.

Group VIII. of O. K. was different to group VII. in the rate of the metronome, which was 120 beats instead of forty, as in the latter. The variations increase here also with the increase in the normal angles. The valuations agree with those in group VI. At 15° and 30°, there are undervaluations of .04° and 1.19°, and at 60° an overvaluation of .27°. In connection with the increase of compared angles as compared with that of normal ones, groups V. and VI. agree with groups VII. and VIII. This agreement corresponds with the variations in the rate of movement. The thresholds show here also an increase in the absolute difference sensibility with increase in the normal angles. They are respectively .40°, .56° and 1.09° for angles 15°, 30° and 60°. The relative difference sensibility increases gradually with the increase in the normal angles. Groups VII. and VIII. show that the valuation is more exact at the rates of forty beats of the metronome than at the rates of 120, and this from three standpoints, namely, that of the valuation as determined by $\triangle$, that of the absolute difference sensibility, and that of the relative difference sensibility determined by $\frac{\Delta r}{r}$.

Observer Ch. Groups I., II., III. and IV. of O. Ch. were carried out similarly to groups VI., VII., VIII. and IX. of O. K. Group I. consists of upwards movements, etc. The variations are in this group especially large compared with the same in the groups so far considered. They vary between 1.02° and 2.66°, and are much larger at 60° than at 15° and 30°. The valuations are undervaluations. At 15° and 30°, they are .43° and .09°, the latter an approximately correct valuation. The compared angles at 15° and 60° are more undervalued in comparison with their normal angles than at 30°. The threshold at 30° is smaller than at 60°, and at 60° smaller than at 15°. The absolute difference sensibility follows this order also. The thresholds are for 15°, 30° and 60°, respectively, 1.14°, .76° and .85°. The relative difference sensibility increases more between 15° and 30° than between 30° and 60°.

Group II. of O. Ch. varies from group I. only in so far as the rate is 120 instead of forty beats per minute. The variations vary between .44° and .75°, and are somewhat greater at 15° and 30° than at 60°. The valuations are at 15° and 60° overvaluations, namely, .26° and 1.35°, respectively. At 30° there is an undervaluation of .18°. The relation of changes of the compared angles to their normal ones is the reverse of what they were in group I., where the normal angles vary. The thresholds are greater than in all the groups considered above. They are for 15°, 30° and 60°, respectively, 1.48°, 1.71° and 3.15°. Hence the absolute difference sensibility decreases with the increase in the normal angles. The relative difference sensibility increases much more between 15° and 30° than between 30° and 60°.

Group III. of O. Ch. consists of downwards movements at angles of 15°, 30° and 60° at a rate of forty beats of the metronome in the minute. The variations follow in their size those of the normal series. At 30° there is an overvaluation of .06°. At 15° and 60° there is an undervaluation of .07°, and an overvaluation of 1.25° respectively. The valuations at 15° and 30° are nearly correct valuations. The thresholds do not vary relatively as do the normal angles. At 15°, 30° and 60° they are respectively 1.00°, .93° and 1.47°, while the absolute difference sensibility is smaller at 60° than at 15°. This last is somewhat less than at 30°. On the contrary, the relative difference sensibility is finer at 60° than at 30°, and finer at 30° than at 15°. The difference between these two is, however, nineteen times greater than that between the first two.

Group IV. has normal angles of 15°, 30° and 60°. The movements were quite similar to those in the above group apart from a change in the rate of forty beats of the metronome to that of 120. △ shows an overvaluation of .21° and .37° at 15° and 60°, and an undervaluation of .19° at 30°. △r gives thresholds of 1.20°, 1.00° and 3.00° at 15°, 30° and 60°, and $\frac{\Delta r}{r}$ shows a higher relative difference sensibility at 30° than at 15° and 60°. The relative difference sensibility is finer at 30° in group III. than in group IV.

Valuations of Passive as Compared with Active Movements.

With O. R. passive movements, as well as active, for the purpose of comparing them with the active, were made in the upwards direction under but towards the horizontal plane. The normal angles were 15° and the rates forty (group I.) and 120 (group II.) beats of the metronome in the minute. The variations of both groups are not large. In group I. they vary between .32° and .44°. In group II. between .29° and .69° △ shows that the valuations on the whole are finer with the passive than with the active movements. In the first there was an overvaluation of only .05° for group I. and of .03° for group II., while for the last there were undervaluations of .23° and .36°. The thresholds in group II. are nearly equal for the active and passive movements, namely, 1.48° and 1.41°. Those in group I. are not nearly so much like one another, since the values are 1.40° and 1.85°. The absolute difference sensibility is finer at the slower rate for the active than for the passive series, and for the quicker rate than for the opposite. The constant size of the thresholds with O. R. are probably dependent on the want of practice, or on his want of sensibility for motion sensations. The relative difference sensibility yields like results to those of the absolute. While now we have so far described the differences between active and passive motions, yet we can now leave them aside, as they are so limited. They serve only to show influences of the rate of movement.

The Relations of Changes in the Rate of Movement.

So far we have considered these experiments on the whole without respect to changes in the rates, which arise within the groups themselves. All the possible rates to be found are as follows: 15° in 1.50 sec. = 1° in .10 sec., 15° in .50 sec. = 1° in .033 sec., 30° in 1.50 sec. = 1° in .05 sec., 30° in .50 sec. = 1° in .016 sec., 60° in 1.50 sec. = 1° in .025 sec., 60° in .50 sec. = 1° in .008 sec., 60° in 3.00 sec. = 1° in .05 sec. If the metronome beats forty times in the minute, each interval is equal to 1.50 sec.; if 120, .50 sec., and if twenty times, 3.00 sec. Applying these figures to the groups we have described, we find that in each series the rate increases at 15°, 30° and 60° in a geometrical progression of one-half. One group of motions with 15°, 30° and 60° as normal angles and a rate of forty beats in the minute, implies relative rates of for 15°, 30° and 60°, 1° in .100 sec., 1° in .05 sec. and 1° in .025. Now these increase in the relation of a geometrical progression of one-half to one another. Where the metronome beats 120 times in the minute, the corresponding relative rates are 1° in .03 sec., 1° in .016 sec., 1° in .008 sec. These stand to one another in the relation of a geometrical progression of one-half. I presuppose here that if the metronome beats forty times a minute, the movements last the length of time between the two beats. If this is not exactly so, still the variations from this at

15°, 30° and 60° is nearly alike in each case and very small, and moreover these relative rates show the exact duration of the movements.

On the Relation between Stimuli and Difference Sensibility.

The sizes of the normal angles themselves bear to one another the relation of a geometrical progression of one-half, and the rates also bear the same relation to one another. This is true, also, although the duration of movements for the different normal angles remains the same. If we consider the size of these angles, with the rates of movement as stimuli, then we have in each group, considered above, three stimuli, which bear to one another the relations of parts of a geometrical progression. Now, Weber's law says that in order to let the rate of movement increase in an arithmetical progression, the stimuli must increase in a geometrical progression. Of the eight groups of O. K. there are only four where this law holds approximately, namely, in groups III., IV., VII. and VIII. In all the groups the direction is towards the horizontal plane; III. and IV. upwards, from below, towards the horizontal plane; VII. and VIII. downwards from above, towards the same. As measure of the sensibility I have taken $\frac{\Delta r}{r}$. When one reckons out this sensibility value to the tens of thousands, it is sufficiently approximate without reckoning out $\frac{\Delta r_0 + \Delta r_x}{2}$. The values obtained by this last vary only five to ten thousand parts from those obtained by the $\frac{\Delta r}{r}$. In group III., O. K., the difference sensibility for 15°, 30° and 60° respectively, is represented by 226, 160 and 108. The differences are, therefore, sufficiently approximate to an arithmetical mean of 60. In group IV. the arithmetical mean is about 57, in group VII. about 60, in group VIII. about 65. In these four groups Weber's law appears to hold. In the other four groups of O. K. the changes in sensibility do not follow any distinct law. In group I. the sensibility at 60° is nearly three times finer than at 15°, while the difference between that at 30° and 60° is nearly four times greater than that of the same between 15° and 30°. This difference might easily come about through habitual movements in this direction and practice. In group II. the sensibility is in general not great. That it is greater at 15° than at 30° can be explained through habit. In group V. the small sensibility at 60° prevents Weber's law from being applicable to the results. These motions at 60° are uncustomary. In group VI. the difference sensibility is finer at 30° than at 15° and 60°. I see no explanation of the variations in the sensibility in this group. In the ordinary movements which one makes, the direction is towards the horizontal plane. Goldscheider[1] has shown that sensations of locality exist by which one can recognize the place where the arm is held. In the ordinary movements towards the horizontal plane, these sensations of locality are better developed. The more regular sensibility for different distances in motion towards this plane can be influenced through this. Only in group III. of O. C. is there any approximate application of Weber's law. The difference between the values for sensibility at 15° and 30° is 183, between those at 30° and 60°, 107, an approximate arithmetical mean of 145. In group IV. the sensibility is throughout not fine. That at 15° appears, however, to be finer than that of the same

[1] "Ueber den Muskelsinn." Du Bois-Reymond's Archiv, 1889 and Appendix.

observer for other directions and rates of movement. The very small sensibility which O. C. shows in comparison with O. K. throughout is worthy of attention. The differences between the sensibilities at 15° and 30° and 30° and 60°, in groups I. and II. of O. C., are peculiar, in so far as in both cases the difference between the first two is much finer than that between the last two. Between 30° and 60°, in group I., the sensibility increases 2.29 times more than between 15° and 30°, and in group II. 3.48, times. With O. Ch. Weber's law does not hold at all. In group I. the difference between 15° and 30° is 4.5 greater than that between 30° and 60°, and in group II. nine times. It appears in groups I. and II. of O. C. and O. Ch., that while Weber's law does not hold, a law of another kind holds. This appears to be dependent on the rate of movement in so far as the tripling of the rate of movement doubles the relation of the differences in the sensibility of either of the observers. In the case of the first observer 9 is twice as great as 4.5, and in that of the last, 3.48 is 1½ times as great as 2.26. This doubling of the relations of the differences to one another is accompanied by a triple increase in the rates of movement for both observers. For 15°, 30° and 60° the rates are respectively—in group I., 1° in .10 sec., 1° in .05 sec. and 1° in .025 sec., and in group II., 1° in .033 sec., 1° in .016 sec. and 1° in .008 sec. The groups of O. K., where the deviations from Weber's law are found, namely, I., II., IV. and VI., correspond to these groups. The differences in these groups agree, however, only partly with the results of O. C. and O. Ch. In group III. 4 is twice as great as 2 in group I. (the relative values for the relation of the differences). Group VI., however, yields no value to be compared with 4 in group V. Series III. and IV. of O. Ch. allow of no application of Weber's law which the corresponding groups of O. K. show. The very fine difference sensibility at 30°, in comparison with that at 15° and 60°, explains this partly. It appears, then, that movements towards the horizontal plane, as, *e. g.*, III., IV., VII. and VIII of O. R., III. and IV. of O. C., and III. and IV. of O. Ch., with exceptions, follow Weber's law, while such as have a direction of movement away from the horizontal plane, as, *e. g.*, groups I., II., V. and VI. of O. K., I. and II. of O. C. and O. Ch., give greater differences of difference sensibility between 15° and 30° and 30° and 60° than Weber's law presupposes. The peculiarities in the different kinds of movements, then, are dependent as well on the normal lengths as on the rates of movement, and these latter are probably influenced by the direction of movement.

The Relations of the Valuation to One Another.

The table given on page 402 shows the values for △, for the different directions and groups of O. K., O. C. and O. Ch. I. to IV. show it for motions which were made under the horizontal plane, and V. to VIII. for such as were above the same, a and b represent rates of respectively forty and 120 beats of the metronome in the minute. U and D show whether the movements were up or down. The symbols minus and plus, when they are enclosed in brackets, show whether there was an over or undervaluation. When the plus and minus stand outside of these, they show the relative valuations of the three normal angles, 15°, 30° and 60°, to one another. The values for the three normal angles are given from left to right for angles of 15°, 30° and 60° in each diagram. I., III., V. and VII. are upwards movements, II., IV., VI. and VIII. downwards.

TABLE III.

U.					D.				
		15°	30°	60°			15°	30°	60°
I. C.	a	+(+.26)	—(—.21)	+(—.13)	II. C.)	a	—(—.05)	—(—.04)	+(—.03)
	b	—(—.32)	+(+.34)	—(—.04)		b	+(+.50)	+(+.34)	—(—.31)
III. K.	a	+(+.02	—(—.27)	+(+.27)	IV. K.	a	+(—.04)	—(—.05)	+(—.03)
	b	—(—.68)	+(—.07	—(—.13)		b	—(—.12)	+(+.19)	—(—.81)
V. Ch.	a	—(—.43)	+(—.09	—(—.43)	VI. Ch.	a	—(—.07)	+(+.06)	+(+1.25)
	b	+(+.26)	—(—.18)	+(+.35)		b	+(+.21)	—(—.19)	+(+.37)
VII. K.	a	+(+.12)	—(—.16)	—(—.26)	VIII. K.	a	+(+.05)	—(—.10)	+(0.00)
	b	—(.07)	+(+.15)	+(+.56)		b	+(—.04)	—(—.19)	+(+.27)

It results that the best valuation for downwards movements was at the rate of forty beats a minute, namely, in groups a II., a IV., a VI. and a VII. There is almost a correct valuation by each observer for all the angles. A single exception to this is at 60°, by O. Ch. The valuations of the other movements vary between an overvaluation of .56° and an undervaluation of .81°. In only b III. and a V. is there an undervaluation for all three normal angles. In all the other groups there are more or less overvaluations as well as undervaluations. In no group are there undervaluations alone. Apart from groups b III. and a V. it is, then, to be assumed that if movements had been made at the angles lying between the angles used as normals, with similar rates of movement (forty or 120 beats), correct valuations would have resulted. One finds, further, from the tables, with few exceptions, an overvaluation where the metronome beats forty times a minute, and inversely an undervaluation where it beats 120 a minute for normal angles of 15° and 60°. At 30° the reverse holds true. From this it is further to be assumed that if one investigated the angles between the normal angles with both rates of movement, several such normal angles between 15° and 60° would be found for which correct valuations would be given.

That at the two rates of movement for the same normal angles and movements, the relative valuations are the inverse to one another, is shown in the tables by the plus and minus symbols marked outside of the brackets. The relations of the over and undervaluations and time of the changes in the valuations are not the same in all the groups. In the first groups, namely, for motions under the horizontal plane, there is for angles 15° and 60°, and at the slow rates of movement, a relative overvaluation, and at 30° a relative undervaluation, at the quicker rates of movement for 15° and 60° inversely a relative undervaluation, and at 30° a relative overvaluation. Group II. shows some exceptions to this. The relative over and undervaluation signify nothing else than that in one case the angles are more highly valued at 15° and 60° than at 30°; in the other case, less. Group II. shows that the smaller angles at the slower rate are less valued than the larger ones. When the rate is quicker the reverse holds. When the motions were made above the horizontal plane, the relations of the valuation change alternately with the groups. Still the rates of movement show

their peculiar inverse changes in the valuations. In groups V. and VI. the angles are less valued than at 30°, where the slow rate of movement is used. Where the quick rate is used the reverse is true. In group VII., at the slow rate, the smaller angles are higher valued than are the larger, and at the quicker rate the opposite is true. Group VIII. shows that the angles at 15° and 60° are higher valued at the slow rate of movement than those at 30°, and at the quicker rate less valued at 15° and 60° than at 30°.

The Steadiness of the Motions.

The downwards movements appear on the whole to be valued with less certainty than the upwards. The average of all the variations of the groups of the latter for O. C. is .62°, and the same of the downwards .795°. The average of all the former of O. K. under the horizontal plane is .4009°, the same of the latter .684°. Above the horizontal plane the corresponding figures are .35° and .40°. The groups of O. Ch. give an opposite result in so far as the average of the variations of the upwards motions is 1.14° and that of the downwards .53°. It is worthy of remark that the upwards movements of this observer were less exact where the rate was slow than where it was quick, as the variations of the former ranged between 1.02° and 2.66°, and those of the latter between .44° and .75°. In so far as the average of the variations was .46° at the slower rate and .59 at the quicker of the downwards movements, the reverse holds true in regard to them. This observer could only with difficulty execute the larger normal angles at the quicker rate for experimental purposes, and his motions appear more as reflexes than as regulated movements.

TABLE IV.

U.					D.				
		15°	30°	60°			15°	30°	60°
I. C.	a	613 +(+)	430 −(−)	323 +(−)	II. C.	a	512 −(−)	291 −(−)	194 +(+)
	b	416 −(−)	446 +(+)	393 −(−)		b	833 +(+)	365 +(+)	229 −(−)
III. K.	a	226 +(+)	160 −(−)	108 +(+)	IV. K.	a	280 +(+)	233 −(−)	99 +(−)
	b	283 −(−)	240 +(−)	208 −(−)		b	280 −(−)	351 +(+)	219 −(−)
V. Ch.	a	760 −(−)	253 +(−)	141 −(−)	VI. Ch.	a	666 −(−)	145 +(+)	291 +(+)
	b	986 +(+)	570 −(−)	525 +(+)		b	806 +(+)	333 −(−)	500 +(+)
VII. K.	a	246 +(+)	196 −(−)	186 −(−)	VIII. K.	a	246 +(+)	176 −(−)	125 +(0.00)
	b	373 −(−)	733 +(+)	198 +(+)		b	266 +(−)	186 −(−)	136 +(+)

In tables similar to the above I have put in the values for the relative difference sensibility. In groups II., III., VI., VIII. and V., and for the most part also in groups I. and VII., it appears that the

movements are executed with finer difference sensibility at the slow rate than at the quick. In so far as the relatively smaller rate in any group accompanies the greater angles, it would be desirable to complete these groups so that all the normal angles would have the same rate of movement.

I have completed two other groups below the horizontal plane—one upwards, group II., and one downwards group I.,—in which the rate was kept relatively the same for 15°, 30° and 60°, namely 1° in .033 seconds. For this purpose the metronome beat at 15° 120 times a minute, at 30° sixty times and at 60° thirty times. The O. Hicks was at first somewhat exercised, and afterwards four series of upwards and four series of downwards movements in the order of experimentation were varied alternately with one another. Those for the purpose of exercise were in the downwards direction, and consisted in eight series at respectively 15°, 30° and 60°. These eight series I have reckoned out with the last four similar series, because both the four series and the combined twelve series show similar deviations from the four series of upwards movements.

A very distinct difference between the results of groups I. and II. is present, in so far as $\triangle$ shows that a general overvaluation of the upwards movements and undervaluation of the downwards take place. From this and from the other observers it is to be assumed that the direction of the valuation is dependent on the observer. In group I. $\triangle r$ and $\frac{\Delta r}{r}$ show that increasing the size of the normal angles causes the absolute difference sensibility to decrease and the relative to increase. $\triangle$ shows also that this increase also causes the valuation to be more correct. The variations varied between .28° and .54° and gradually increased with the increase in the normal angles. The difference between the relative difference sensibility, as shown by $\frac{\Delta r}{r}$ at 15° and 30°, is 1.5 times that shown at 30° and 60°.

Group II. of O. H. is the reverse of group I. in so far as $\triangle r$ and $\frac{\triangle r}{r}$ show that increasing the angles decreases the absolute difference sensibility, but increases the relative. This holds for angles of 15° and 60°. At 30°, however, the absolute difference sensibility, as shown by $\frac{\triangle r}{r}$, is finer than at 60°. The values of $\triangle$ do not vary much from one another. The variations Vr_o and Vr_u vary between .12° and .62° and increase with the increase in the normal lengths. The relative difference sensibility is much smaller at 15° than at 30° and 60°. Both series I. and II. show, then, that the increase in the rates with increase in the normal angles produces no great influence upon the increases in the relative sensibility as shown by $\frac{\triangle r}{r}$.

Sense of Locality.

I have called attention to sensations of locality for the arm. O. C. has observed this further in so far as in his upwards movements the starting points instead of the end points were varied. He recognized the changing of the angles in a series by the change in the height to which he must raise or lower his arm in order to reach the starting points. One series of group VII. of O. K. was executed in which the starting point instead of the end point was varied. The valuation and the threshold values are about the same as in

the regularly executed series. From this it appears that the difference in the valuation is not large. It appears, nevertheless, that here an apparently fruitful field for investigation on the sense of locality[1] by the method of just perceptible change, is open. If a height is taken as normal starting point and the thresholds above and below determined, values of the difference sensibility in the sense of locality would be obtained. The values of the average of the heights of the starting points in my first experiments could also be very easily used here, in so far as they show the average variations of the individual valuations of the heights of any assumed starting point.

The results of these angle method experiments by the method of just perceptible change are:

1. The absolute difference sensibility as determined by $\triangle$r for 15°, 30°, 60° is greater at the slower rate of forty beats of the metronome a minute than at the quicker rate of 120 beats.
2. This absolute difference sensibility varies with the observers and with the size of the normal angles.
3. At 30° and 60° it is with few exceptions smaller than at 15°.
4. The relative difference sensibility as shown by $\frac{\Delta r}{r}$ is much finer at the larger normal angles. For motions which are directed towards the horizontal plane, Weber's law holds approximately. For such as are directed away from it, the increase in the relative difference sensibility with increase in the normal angles is much greater than shown by Weber's law.
5. By proper use of different rates it is possible for different observers to execute correctly, according to the values given by $\triangle$, movements at all normal lengths.
6. The absolute difference sensibility as shown by $\triangle$r is finer at quicker rates of movement than that at slower, where the motions are upwards towards the horizontal plane.
7. The difference between the thresholds for motions above and under the horizontal plane is not large for the same observer.
8. Where changes in the size of the normal angles and in the rates take place, the changes in the first produce the greatest influence.
9. The steadiness in the valuations is dependent on the observer. In the case of most observers, valuations of downwards movements are less steady than are those of upwards.

[1] Goldscheider above cited.

TABLE I.—*Downwards One-Handed Movements.*

N. S.	N. E.	H. S.[2]	H. V.	M. R. V.	N. L.	V. N. L.	M. D.	V. D.	R. L.	V. R. L.	M. D. R. L.	V. D. R.	O.
1	9	—53.95	1.80	30	15.97	2.15	1.05	2.95	46.81	2.86	4.50		S.
2	12	—50.00	50.—58.	32	10.02	.86	3.48						S.
3	14	—47.00	2.00		11.30	1.45	.307						S.
4	15	—46.00	84.—59.		16.95	3.48	4.03						K.
5	4	—42.50	2.00	0.00	12.93	2.33	5.87	.68	55.00	8.50	—2.50	13.50	S.
6	17	—32.65	1.68	31½	21.98	1.40	—4.83	2.36	60.77	11.75	5.03	14.56	S.
7	10	—09.00	3.94		18.67	3.68	1.47						K.
8[1]	15	—00.00	+14 +30		10.41	1.43	2.28						K.
9	14	+14.50	3.00		36.06	2.95	—.41						K.
10	5	+22.40	3.94		13.00	1.70	4.40						K.
11	71	+41.70	1.82	30	28.45	3.09	.77	3.38	67.50	6.54	—6.18	12.25	S.
12	9	+46.67	3.26	25	37.61	3.42	3.00						S.
13	8	+49.98	4.00	00	34.47	5.02	—1.63						S.
14	8	+56.07	4.81	46	16.62	2.22	2.28						W.
15	24	+58.59	1.20		24.82	2.67	8.85						K.
16	48	+63.00	4.00		59.98	9.74	3.45						S.
17	30	+63.50	4.01		27.97	11.67	2.86						K.
18	23	+70.00	2.00—3.00—		9.98	.91	.57						S.
19	12	+70.37	.85	59	12.41	1.40	4.14						S.
20	15	+72.00	+72 —22	85¾	6.75	.70	1.21						K.
21[2]	10	+50.00			19.42	3.07	1.82						S.

[1]Observer sat. [2]Made by shadow method.

N. S., number of the series. N. E., number of experiments in each series. H. S., average height of starting point for each series. H. V., average variation of same. M. R. V., as described above. N. L., average length of normal curves of each series. V. N. L., average variation of same. M. D., average difference between normal and compared curves of each series. V. D., average variation of same. R. L., average length of radii of normal curves. V. R. L., average variation of same. M. D. R. L., average difference between radii of normal and compared curves. V. D. R., average variation of same. O., observer.

TABLE II.—*Upwards One-Handed Movements.*

N. S.	N. E.	H. S.[2]	H. V.	M. R. V.	N. L.	V. N. L.	M. D.	V. D.	R. L.	V. R. L.	M. D. R. L.	V. D. R.	O.
1	9	—59.28	1.15	90	21.25	1.88	6.77						S.
2	18	—58.73	.91	52	19.83	4.35	8.29	4.35	47.33	?	7.90	8.55	S.
3	15	—55.00	38.72	38¾	13.95	2.05	6.66						S.
4	15	—54.97	2.33	33	16.93	1.73	3.30						S.
5	12	—54.67	1.24		19.54	2.38	2.54	2.18	50.91	3.83	9.75	4.79	S.
6	10	—53.70	1.80	48	16.50	1.55	4.50	2.10	59.10	12.28	10.50	16.44	S.
7	15	—51.17	1.83	45¼	35.45	4.68	9.80	3.46	57.66	5.39	2.83	6.20	S.
8	15	—48.34	1.37	39	14.15	1.92	3.91						S.
9	17	—41.92	2.39	55	41.96	6.28	12.10	7.46	68.58	6.32	4.94	8.19	S.
10	8	—36.63	3.62	18¾	36.68	4.29	6.68						S.
11[1]	15	—34.00	3.72		18.91	1.85	4.21						K.
12[1]	11	—33.00	2.26		10.81	2.30	.51						K.
13	23	—32.20	1.70	93	29.73	1.69	13.91						S.
14	40	—23.59	3.04		14.10	1.53	2.93						S.
15	8	—22.25	4.31	43¾	32.53	2.29	1.81						S.
16	4	—16.13	3.31	75	10.68	1.95	2.31						S.
17	12	—15.70	3.71	54	26.33	2.22	4.86	2.46	63.75	9.21	.87	10.45	S.
18	31	—12.10	4.25		14.27	1.92	5.43						S.
19	15	—10.20	2.13	45¼	48.03	5.17	4.65						K.
20	22	— 6.30	4.13		33.33	6.17	5.80						S.
21	14	— 4.93	3.14	71½	20.78	1.69	10.18						S.
22	15	+14.25	2.68	30	23.75	1.42	4.89						S.
23	16	+15.60	3.19	51	19.54	3.53	5.31						S.
24	13	+17.35	2.12	21½	13.82	1.54	6.19						S.
25	15	+18.33	2.03	61½	14.78	2.47	4.33						K.
26	15	+19.24	2.51	30	15.58	2.10	3.89						S.
27	13	+31.79	1.30	63	15.15	1.69	5.10	2.43	37.54	7.03	9.03	8.15	S.
28	20	+42.00	20.—64.		7.63	1.44	1.00						K.
29	14	+56.79	1.28	44	11.19	1.18	6.07						S.
30	15	+70.00	3.00—5.00		12.20	1.45	.35						S.

[1]Observer sat.

[2]Roughly estimated.